

Document downloaded from the institutional repository of the University of Alcalá: <http://ebuah.uah.es/dspace/>

This is a postprint version of the following published document:

Regadío Carretero, A., Sánchez Prieto & Esteban, L., S. 2019, "Filtering of pulses from particle detectors using neural networks by dimensionality reduction", Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, vol. 942, art. no. 162372.

Available at <https://doi.org/10.1016/j.nima.2019.162372>

© 2019 Elsevier

(Article begins on next page)



This work is licensed under a

Creative Commons Attribution-NonCommercial-NoDerivatives
4.0 International License.

Filtering of pulses from particle detectors using neural networks by dimensionality reduction

Alberto Regadío^{a,*}, Sebastián Sánchez-Prieto^b, Luis Esteban^c

^a*Electronic Technology Area, Instituto Nacional de Técnica Aeroespacial, 28850 Torrejón de Ardoz, Spain*

^b*Department of Computer Engineering, Space Research Group, Universidad de Alcalá, 28805 Alcalá de Henares, Spain*

^c*Universidad Antonio de Nebrija, C/Pirineos, 55 28040 Madrid, Spain*

Abstract

This article presents a comparison between different filtering methods based on dimensionality reduction for pulses generated on particle detectors. This reduction has been carried out using Neural Networks (NNs). In particular, three topologies have been used: Autoencoders (AEs), Denoising Autoencoders (DAEs) and Restricted Boltzmann Machines (RBMs). A detailed explanation of these methods, a noise reduction analysis, filtering with simulated data and processing of pulses from a neutron detector have been carried out to verify the feasibility of using these NNs as pulse filters.

Keywords: Digital pulse processing, Pulse filtering, Noise, Dimensionality reduction, Denoising, Autoencoder, Restricted Boltzmann Machine, Neural Network

1. Introduction

When particles interact with particle detectors, pulses of current or charge are generated. In general, they are converted to voltage pulses at the output of the preamplifying stage and analyzed in successive stages. The features of these pulses carry several useful information about the incident particle such as its type, energy, or angle of impact [1].

Output pulses of particle detectors are always mixed with noise that limits the accuracy of the information carried by them. In these systems, noise is mainly generated in the detector and in the readout circuit attached to the detector. Some of the noise is a result of fundamental physical processes such as the discrete nature of electric charge and therefore cannot be avoided. However, its effect can be reduced by implementing proper noise filtering strategies with analog and digital electronics.

Compared to analog filters, digital filters reduce the number of electronic devices needed. Among them, Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) are the most common in particle detectors [1]. In addition, they allow to implement more complex shapers and filters to improve the incoming

*Corresponding Author

Email addresses: regadioca@inta.es (Alberto Regadío), sebastian.sanchez@uah.es (Sebastián Sánchez-Prieto), lesteban@nebrija.es (Luis Esteban)

29 pulse quality. Examples of these methods include Lagrange interpolations [2], adaptive filtering [3], simu-
30 lated annealing [4], wavelets [5, 6], genetic algorithms [7] and filtering using Singular Value Decomposition
31 (SVD) [8, 9]. This last method is based on dimensionality reduction, concept explained in Section 2.

32 In this article, the dimensionality reduction will be performed using neural networks (NNs), namely
33 Autoencoders (AEs), Denoising Autoencoder (DAEs) and a Restricted Boltzmann Machines (RBMs), which
34 will be explained in detail in Sections 3 and 4, respectively. This new approach involves advantages that
35 will be explained throughout the article.

36 The use of NNs on particle detectors is not new. They have been used for gamma/neutron [10] and
37 alpha/gamma discrimination [11], particle identification coming from cosmic rays [12, 13] and nuclide iden-
38 tification [14–16]. The cited articles have in common that NNs, namely Adaline or Multilayer perceptrons,
39 allow the identification of particles. However, in this paper, we propose their use to maximize the Signal-
40 to-Noise Ratio (SNR), leaving the identification for later stages of the detection chain of the spectroscopy
41 system.

42 The proposed filter isolates the complete pulse before being processed. Therefore, the proposed filters
43 does not add of ballistic deficit with respect to the previous stages and is feasible for any type of detector.
44 In other words, with FIR or IIR filters we cannot transform CR–RC pulses into CR pulses without ballistic
45 deficit unless filters are used to isolate the pulse.

46 The rest of the paper is structured as follows: Section 5 shall be devoted to compare the filtering methods
47 used in this paper. The results of applying these methods with simulated and real pulses are exposed in
48 Section 6. Finally, the conclusions of this work close the manuscript.

49 **2. Dimensionality reduction and manifold learning**

50 A concept underlying dimensionality reduction is that of a manifold. Mathematically, it is a connected
51 region embedded into a space that locally can be approximated by another space of lower dimension. A
52 typical example of manifold is the Earth. We experience the surface of our planet as a two-dimensional
53 plane, but it is actually a sphere embedded into a three-dimensional space. The definition of locality of each
54 point of the manifold implies the existence of transformations that can be applied to move on the manifold
55 from one position to a neighboring one. In the example of the Earth surface as a manifold, we can walk
56 along the four cardinal points [17].

57 The formal definition of manifold add several constraints but it is part of the differential geometry and
58 is out of the scope of this paper. In NNs, it tends to be used more loosely to designate a connected set
59 of points that can be approximated well by considering only a small number of dimensions embedded in a
60 higher-dimensional space. Each dimension corresponds to a local direction of variation.

61 AEs, DAEs and RBMs are subsets of NNs that carry out manifold learning by dimensionality reduction

62 [18]. They work as follows: given a vector of length (dimension) L , these algorithms assume that most input
 63 combinations $\mathbf{x} \in \mathbb{R}^L$ are invalid, and that valid inputs occur only along a manifold of dimension $H < L$
 64 containing a small subset of points, in this case the pulse without noise. There will be variations in the
 65 output of the learned function occurring only along directions that lie on the manifold. For instance, in
 66 Figure 1, we can see two-dimensional data points that can be grouped onto a one-dimensional manifold.
 67 When the learning process is unsupervised (Figure 1(a)) the manifold is estimated and when the learning is
 68 supervised (Figure 1(b)) the learning data lie on the manifold. Once the NN is trained, the test data points
 69 (x_1, x_2 on the figure) are projected onto the one-dimensional manifold (h_1) and then they are projected
 70 again to (x_1, x_2) (Figure 1(c)).

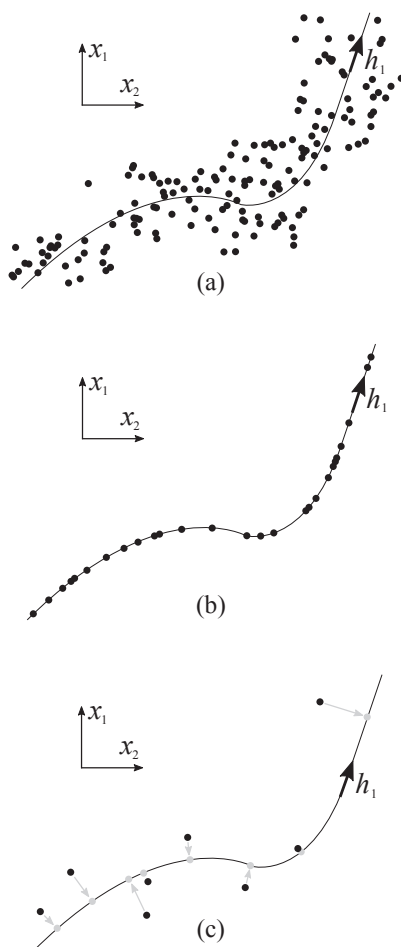


Figure 1: Dimensionality reduction from 2 to 1 (x_1, x_2 to h_1); (a) Unsupervised one-dimension manifold learning; (b) Supervised one-dimension learning; (c) Adjustment of x_1, x_2 using dimensionality reduction (figure adapted from [17])

71 In the context of pulse filtering and along this article, the input dimension L is the length of the pulse
 72 measured in cycles whereas the manifold dimension is H in which we assume that the data lies will be

73 demonstrated along this manuscript. In fact, one key aspect is to find out how many dimensions have
 74 actually the input pulses. As we will see along this article, when pulses with similar shape are reduced to
 75 just one dimension, it is the pulse height.

76 The linearity of the algorithms is related with the fact that the data lie on a linear surface (i.e. a
 77 H -dimensional straight line). However, hereafter all the NNs in this paper will be linear because the vast
 78 majority of the filters used for these applications are also linear [1] and because the optimal weight values
 79 for these NNs can be derived directly by purely linear techniques [19]. Both input and output stages of NNs
 80 will have just one layer because the effect of adding more layers is usually negligible using linear activation
 81 functions.

82 3. Autoencoders (AEs) and Denoising Autoencoders (DAEs)

83 An AE is a type of unsupervised NN that codifies and decodifies an incomplete or noisy input signal
 84 $\mathbf{x} \in \mathbb{R}^L$ with the aim of reconstruct it at the output $\mathbf{y} \in \mathbb{R}^L$. AEs are also defined as NNs that are trained
 85 to learn the identity function [18]. They consist of two parts: an encoder function $\mathbf{h} = f(\mathbf{x})$ that carries out
 86 a dimensionality reduction from L to H and a decoder that produces a reconstruction $\mathbf{y} = g(\mathbf{h})$ restoring
 87 the dimensions from H to L . They have one internal layer $\mathbf{h} \in \mathbb{R}^H$ with H hidden units to store an
 88 autogenerated coding of the input.

89 A diagram of this architecture is presented in Figure 2. Note that constant inputs $x_0 = 1$ and $h_0 = 1$
 90 have been included as a bias at the input and at the hidden layer respectively. Actually, both input and
 91 output layers can be composed by more than one layer but for simplicity these topologies are out of the
 92 scope of this paper.

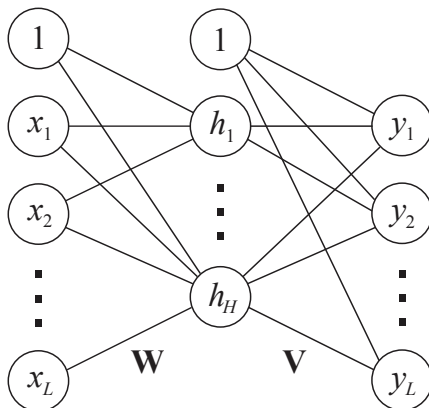


Figure 2: Diagram of a generic autoencoder.

Given an input pulse \mathbf{x} , the value of the hidden layer is

$$\tilde{\mathbf{h}} = \mathbf{W}\mathbf{x} \tag{1}$$

93 where $\mathbf{W} \in \mathbb{R}^{(L+1) \times H}$ is the first weight matrix. The value of H must be lower than L to perform the
 94 dimensionality reduction.

To cover most of the possible space solutions and to try to simulate the behavior of real neurons, an
 activation function should be applied to $\tilde{\mathbf{h}}$:

$$\mathbf{h} = f(\tilde{\mathbf{h}}) \quad (2)$$

95 Common activation functions in NNs include sigmoid, hyperbolic tangent, no activation function ($f(x) =$
 96 x) or the Rectified Linear Unit ($\text{ReLU}(x)$) whose result is x if $x > 0$ and 0 otherwise [20].

97 Once calculated \mathbf{h} , the estimated output value \mathbf{y} is calculated reverting the dimensionality reduction in
 98 the following way

$$\tilde{\mathbf{h}} = \mathbf{V}\mathbf{h} \quad (3)$$

99 where $\mathbf{V} \in \mathbb{R}^{(H+1) \times L}$ holds for the second weight matrix. Then, the same activation function used in (2)
 100 must be applied again to obtain the output \mathbf{y}

$$\mathbf{y} = f(\tilde{\mathbf{h}}) \quad (4)$$

101 There exists a subset of AEs named Denoising Autoencoders (DAEs) that receives noisy data as input
 102 \mathbf{x} and is trained to estimate the noiseless data \mathbf{x}^* as its output \mathbf{y} . The main difference with respect to AEs
 103 is that the learning process of DAEs is supervised.

104 In order to break the linearity, several tests were performed using different activation functions such as
 105 hiperbolic tangent or ReLU. However, one desired feature on filters for particle detectors is linearity and,
 106 in fact, the best results were achieved without using any transfer function, i.e. with linear NNs. For these
 107 reasons, henceforth in this article, we replace (2, 4) by $\mathbf{h} = \tilde{\mathbf{h}}$ and $\mathbf{y} = \tilde{\mathbf{y}}$.

108 3.1. Learning rule

109 Both AE and DAE learn using backpropagation and gradient descent of the cost function j , following
 110 the chain rule for NNs. In learning algorithms, j is defined as as the deviation of the output \mathbf{y} with respect
 111 to the training signal \mathbf{x}^* , that is

$$\mathbf{j} = \mathbf{x}^* - \mathbf{y} \quad (5)$$

According to backpropagation algorithm an the chain rule, when no activation function is used, all entries
 of the weight matrix \mathbf{V} are updated according to the following formula:

$$\mathbf{V} \Leftarrow \mathbf{V} + \alpha(\mathbf{j} \otimes \mathbf{h}) \quad (6)$$

112 where \otimes denotes the external product and α is the learning rate. After applying this equation, the input-
 113 to-inner layer errors δ are calculated applying

$$\delta = \mathbf{V}^\top \mathbf{j} \quad (7)$$

114 where \mathbf{V}^\top is the transpose matrix of \mathbf{V} . This value is used to adjust the weights of the first layer \mathbf{W}

$$\mathbf{W} \leftarrow \mathbf{W} + \alpha(\delta \otimes \mathbf{x}) \quad (8)$$

115 These formulae are repeated for each input during a number of times established by the user to adjust
 116 the weights of the NN.

117 4. Restricted Boltzmann Machines (RBM)

118 RBMs are specific type of Boltzmann Machines and Markov Networks that can learn a probability
 119 distribution over its set of inputs in an unsupervised way.

120 Unlike the Boltzmann machines, RBMs are composed by two groups of units (commonly referred to
 121 as the “visible” and “hidden” groups respectively) set as a bipartite graph (Figure 3). They may have
 122 a symmetric connection between them and there are no connections between nodes within a group. By
 123 contrast, “unrestricted” Boltzmann machines may have connections between hidden units. This restriction
 124 allows for more efficient training algorithms that are available for the general class of Boltzmann machines,
 125 in particular the gradient-based contrastive divergence algorithm.

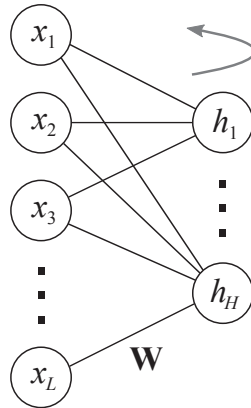


Figure 3: Diagram of a generic Restricted Boltzmann Machine. Bias \mathbf{a}_1 and \mathbf{b}_1 were omitted for clarity.

Given an input pulse \mathbf{x} , the value of the hidden layer is

$$\tilde{\mathbf{h}} = \mathbf{W} \mathbf{x} + \mathbf{a}_1 \quad (9)$$

126 where $\mathbf{W} \in \mathbb{R}^{L \times H}$ is the weight matrix and the \mathbf{a}_1 is the bias.

In the same way that AEs, to cover most of the possible space solutions and to try to simulate the behavior of real neurons, an activation function should be applied to $\tilde{\mathbf{h}}$:

$$\mathbf{h} = f(\tilde{\mathbf{h}}) \quad (10)$$

127 The obtained value \mathbf{y} is

$$\tilde{\mathbf{y}} = \mathbf{W}^\top \mathbf{h} + \mathbf{b}_1 \quad (11)$$

128 where \mathbf{W}^\top is the transpose matrix of \mathbf{W} and \mathbf{b}_1 is another bias, generally different from \mathbf{a}_1 .

129 Finally, the same activation function must be applied again to obtain the output \mathbf{y}

$$\mathbf{y} = f(\tilde{\mathbf{y}}) \quad (12)$$

130 In order to keep the linearity of the network and in the same way that with AEs and DAEs, we replace
131 (10, 12) by $\mathbf{h} = \tilde{\mathbf{h}}$ and $\mathbf{y} = \tilde{\mathbf{y}}$.

132 4.1. Learning rule

133 In the same way that AEs, RBMs learns using backpropagation and gradient descent in an unsupervised
134 way. However, in this case the output comes back from the hidden units, as shown in Figure 3, that yields

$$\mathbf{z} = \mathbf{W}\mathbf{y} + \mathbf{b}_1 \quad (13)$$

135 Then, if we define \mathbf{P} and \mathbf{N} as

$$\begin{aligned} \mathbf{P} &= \mathbf{h} \otimes \mathbf{x} \\ \mathbf{N} &= \mathbf{z} \otimes \mathbf{y} \end{aligned} \quad (14)$$

the weights and bias become

$$\begin{aligned} \mathbf{W} &\Leftarrow \mathbf{W} + \alpha(\mathbf{P} - \mathbf{N}) \\ \mathbf{a}_1 &\Leftarrow \mathbf{a}_1 + \alpha(\mathbf{x} - \mathbf{y}) \\ \mathbf{b}_1 &\Leftarrow \mathbf{b}_1 + \alpha(\mathbf{h} - \mathbf{z}) \end{aligned} \quad (15)$$

136 Similarly to AEs and DAEs, these formulae are repeated for each input during a number of times
137 established by the user to adjust the weights of the NN.

138 **5. Comparison among methods**

139 In this section, AEs, DAEs and RBMs are compared to SVD filtering [9]. The reason for this comparison
 140 is that these methods share common features: (a) the pulse must be isolated in an input \mathbf{x} ; (b) both NNs
 141 and SVD filtering are linear; (c) all of them can be used for dimensionality reduction as explained in Section
 142 2.

143 SVD is a linear procedure that transforms a set of N observations \mathbf{x} of length L grouped in $\mathbf{X} \in \mathbb{R}^{N \times L}$
 144 to a new coordinate system in which the value of the first coordinate have the largest possible variance, and
 145 the values of each succeeding coordinates have the largest possible variance under the constraint that they
 146 are uncorrelated with the preceding coordinates [18]. Thus, \mathbf{X} is decomposed according to the following
 147 formula:

$$\mathbf{X} = \mathbf{U}\mathbf{S}\mathbf{T}^\top \tag{16}$$

148 where $\mathbf{S} \in \mathbb{R}^{N \times N}$ is a diagonal matrix and $\mathbf{U} \in \mathbb{R}^{N \times N}$ and $\mathbf{T} \in \mathbb{R}^{L \times N}$ are orthonormal matrices.

149 The alternative basis of dimension H is obtained by selecting the higher H eigenvalues of \mathbf{S} and by
 150 replacing the other by zeros. Afterwards, the inverse change of basis is applied to obtain the filtered pulse in
 151 the original basis multiplying the modified \mathbf{S} by \mathbf{T}^\top . All of these operations can be boiled down to a single
 152 matrix: the SVD filtering matrix. A feature of this matrix is that when calculated with noiseless pulses,
 153 its eigenvectors are the noiseless pulses with unitary height and its eigenvalues their height. A detailed
 154 description about this method and its performance can be found in [9].

155 As explained in Section 2, the components of this basis are transformed according to a new basis with a
 156 lower dimension and afterwards transformed back. These two transformations are similar to the one carried
 157 out in AEs and DAEs using \mathbf{W} and \mathbf{V} . However, unless NNs, in SVD filtering the reduced basis represented
 158 by \mathbf{U} and \mathbf{T} are orthonormal and therefore yields different results.

159 Actually, the main advantage of implementing NNs instead SVD is the less use of memory and the lower
 160 computation time during the pulse processing, namely $N \cdot L$ in SVD compared to $2 \cdot L \cdot H$ in NNs (note
 161 that in general $H \ll N$). An additional advantage is that a NN can be re-trained with new pulses when
 162 necessary.

163 About the NN types, both DAEs and AEs use separate matrices to perform the two changes of basis
 164 whereas RBMs use only one matrix. This fact allows RBMs to have a faster convergence in weights but also
 165 it can affect their performance as we will see in the next section.

166 AEs and RBMs are unsupervised whereas DAEs are supervised. In fact, DAEs transform the input
 167 pulse to another. This is particularly useful in DAEs to avoid the modification of the shape of the pulses
 168 as a consequence of the degeneration of the detector due to radiation (e.g. in space environment). Hence

169 by using DAEs, the pulse filters can be recalibrated. Similar methods to avoid this issue were discussed in
 170 [21–24] for specific shaping types such as trapezoidal and cusp-like.

171 On the contrary, the use of NNs compared to the use of SVD filtering has the following drawbacks: (a)
 172 high computation time during the training; (b) need for the NN to converge. In every test executed by
 173 setting the learning rate α to an adequate value the NNs converged properly.

174 6. Results

175 In order to validate the performance of the filters based, a set of simulations and tests have been carried
 176 out. These filters can be placed after the Analog-To-Digital Conversion (ADC) or after the digital shaping
 177 stage. In these tests, the former option was chosen.

178 6.1. Results with simulated pulses and noise

179 In this test, a set of 2500 CR–RC pulses with arbitrary heights and white noise were generated as
 180 a training set for the NNs. Afterwards, another subset of pulses of the same type were generated for
 181 backtesting the trained NNs. Examples of filtering using each method are shown in Figure 4. Clearly, the
 182 results are improved with respect to an example that uses a FIR filter with $y[z] = \frac{1}{10} \sum_{k=1}^{10} z^{-k}$ as response
 183 function (Figure 5). As explained in the previous section, SVD filtering has been also applied to the same
 184 pulses for comparison purposes.

185 In figure 4 we can observe in all cases that using a low value of H we obtain the best results. It is also
 186 observed that the DAE outperforms the AE and RBM which is to be expected since the DAE is trained
 187 with \mathbf{x}^* instead \mathbf{x} .

To see in more detail the deviation of the output signal with respect to the ideal one we considered Pulse
 Height Analysis (PHA) and Pulse Shape Analysis (PSA) because they are two of the most common ways
 of extraction of information from pulses. Then, given an ideal pulse \mathbf{x}^* , the error Δ_H is defined for PHA as

$$\Delta_H = |\max(\mathbf{x}^*) - \max(\mathbf{x})| \quad (17)$$

Whereas for PSA, the error is defined as

$$\Delta_S = \frac{1}{L} \sum_{k=0}^L |\mathbf{x}^* - \mathbf{x}| \quad (18)$$

188 The error magnitude vs. H for PHA and PSA is depicted in Figure 6. In this figure, we verified that
 189 the lower the value of H , the better the noise is filtered. This fact fits into the premise exposed in section
 190 2 that the first dimension of the pulses is their height. Only when the shape of the training pulses becomes
 191 very different from each other the unfiltered noise stops being proportional to H as will be shown in the
 192 next section.

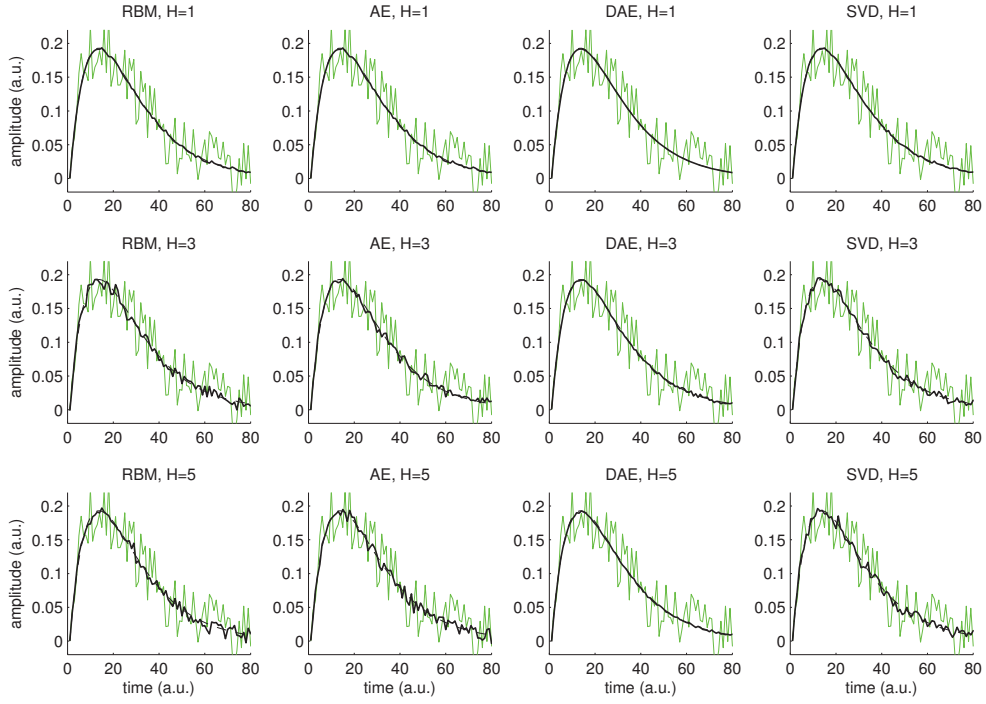


Figure 4: Filtering pulse comparison. The green line is the input pulse \mathbf{x} , the black dashed line represents the objective output \mathbf{x}^* and the black solid line is the obtained output \mathbf{x} .

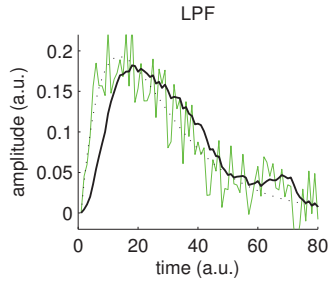


Figure 5: Low-pass filtering.

193 The lowering of the cost function \mathbf{j} as the epochs go on for the three NNs is shown in Figure 7. Along
 194 this work the learning rate $\alpha = 0.1$. We can observe that RBM is fastest in its convergence followed by AE
 195 and DAE. The results also indicate that H does not affect the speed of convergence too much. Obviously,
 196 the more training pulses are used the lower \mathbf{j} are achieved.

197 About the shape of the incoming pulses (e.g. replace in training and tests CR–RC by a triangular shape),
 198 no changes in the efficiencies of the pulses are observed although Δ_H and Δ_S oscillate slightly depending on
 199 the shape of the pulse. In other words, DAE continues to be the best, followed by AE and RBM, while SVD

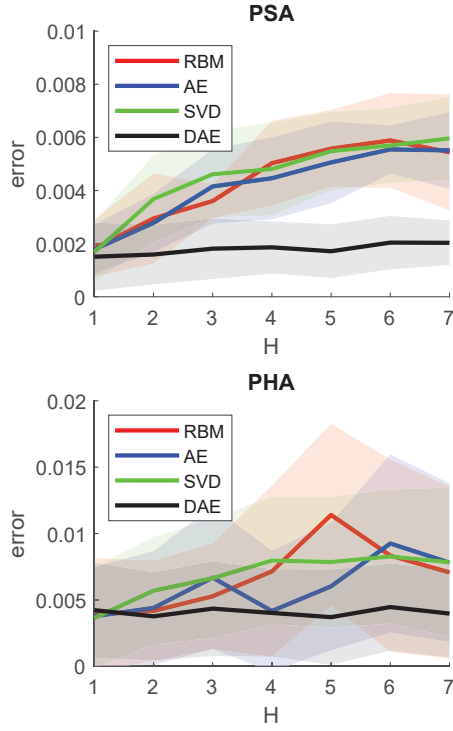


Figure 6: Δ_S and Δ_H with their corresponding standard deviation vs. H for PHA and PSA.

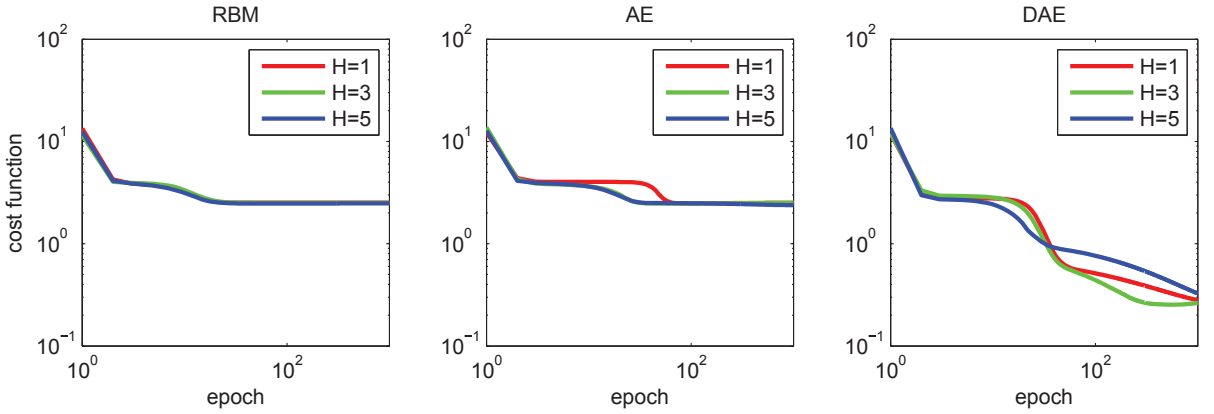


Figure 7: Learning rate for different values of H . The cost function stands for the quadrature sum of each component of \mathbf{j} .

200 filtering offers slightly worse results. What is still observed is that according to the general rules [25], the
 201 effect of white noise is inversely proportional to shaping time τ_s and Brownian noise is directly proportional
 202 to it.

203 *6.2. Noise filtering analysis*

204 In this section, we will analyze the noise impact on the NNs used in this article. Such impact is propor-
 205 tional to the Equivalent Noise Charge (ENC) Q , which is the quadrature sum of the spectral density of each
 206 noise type multiplied by its corresponding system's response to that type of noise [26].

207 When calculating the impact of the two fundamental types of noise that exist in all low-noise pulse
 208 amplifiers, voltage (white) noise and current (brownian) noise [25], and normalizing the detector capacitance
 209 (i.e. $C = 1$ F), we get

$$Q^2 = i_n^2 F_i \tau_s + v_n^2 F_v \tau_s^{-1} \quad (19)$$

210 where i_n^2 is the current noise spectral density measured in $A/\sqrt{\text{Hz}}$ and v_n^2 is the voltage spectral density in
 211 $V/\sqrt{\text{Hz}}$, $\tau_s = T_s L$ (T_s the sampling period) holds for the duration of the pulse, F_i and F_v are the squared
 212 system's response (also called noise index) $h(t)$ for a unit of current and voltage noise, respectively. For
 213 Linear Time-Invariant (LTI) systems, a unit of voltage noise is a Dirac delta function $\delta(t)$ and a unit of
 214 current noise is a step function $u(t)$ [25]. Thereby, taking into account that $h(t) * \delta(t) = h(t)$, the system's
 215 response to noise are equal to

$$F_i = \frac{1}{S^2} \int_0^\infty (h(t) * u(t))^2 dt \quad (20)$$

$$F_v = \frac{1}{S^2} \int_0^\infty (h(t))^2 dt \quad (21)$$

216 where S is the signal amplitude. Along this paper, $S = 1$ because it is assumed that pulses will be filtered,
 217 not amplified.

218 Noise spectral densities (i_n and v_n) are specific of each particle detector type and can be estimated
 219 using thermal and shot noise models among others [27]. However, any accurate calculation of noise spectral
 220 densities demands a detailed knowledge of the physical processes involved in the circuit elements. In contrast,
 221 F_i and F_v are completely determined by the shaper or filter response [25] and for this reason we focus on
 222 them along this Section.

223 In addition, system's response for generalized noise types can also calculated using fractional calculus
 224 [28, 29]. This technique allows to obtain additional noise responses such as $1/f$ -noise [30]. One way to
 225 obtain the fractional derivative or integral of the Dirac delta function $\delta(t)$ centered at t_0 is obtained via the
 226 Riemann-Liouville definition [28, p. 106]. Thus, we define ν_β as

$$\nu_\beta \equiv D^\beta \delta(t - t_0) = \begin{cases} \frac{(t-t_0)^{-\beta-1}}{\Gamma(-\beta)} & \text{if } t \geq t_0 \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

227 where $\Gamma(\cdot)$ is the Gamma function and $D^\beta \equiv \frac{d^\beta}{dt^\beta}$ is the differential operator. In this way, for instance when
 228 $\beta = 1$ we are working out the first derivative and when $\beta = -1$ the first integral.

229 Thus, Eq. (20, 21) can be generalized to

$$F_\beta = \frac{1}{S^2} \int_0^\infty (h(t) * \nu_\beta(t))^2 dt \quad (23)$$

230 Thus, for example, for $1/f$ -noise, the filter response will be equal to $F(0.5)$. Eq. (20, 21, 22) are
 231 applicable to both analog and digital filters. Figure 8 shows some values of the discretized ν_β depending on
 232 the β applied. Note that noise types with $\beta < -1$ generate infinite $F(\beta)$ unless bipolar pulse shapers are
 233 included [30].

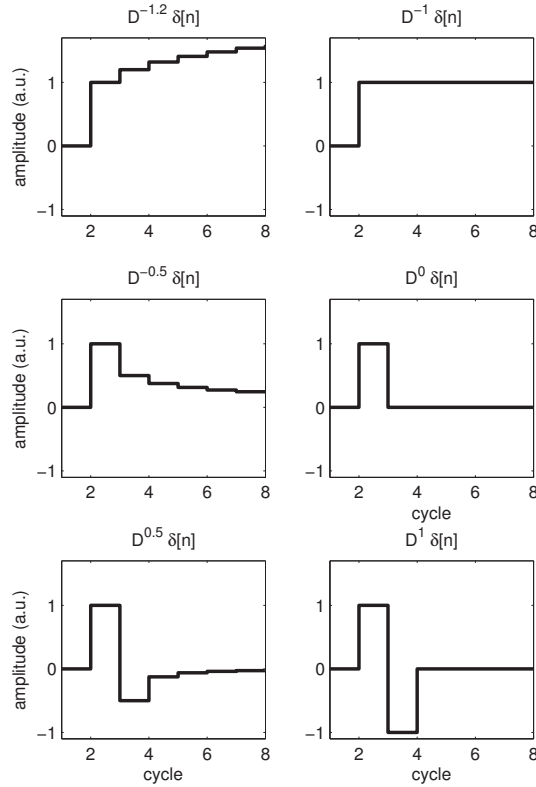


Figure 8: Some differintegrals of the discrete Dirac function. $\nu_0 = D^0 \delta[n]$ is the unit of voltage noise (discrete delta function), $\nu_{-0.5} = D^{-0.5} \delta[n]$ is the unit of $1/f$ noise, and $\nu_{-1} = D^{-1} \delta[n]$, that is its integral, is the unit of current noise.

234 However, noise indexes of shapers and filters based on isolation of pulses (namely NNs) are not comparable
 235 among them. This is because we distinguish between shapers, which change the shape of the pulse with the
 236 aim of reducing noise, and filters, which reduce noise leaving the signal unaltered. For the latter, we define
 237 the ratio of a given noise type as the quotient between the effect of the filtered noise and the unfiltered noise.

$$N_\beta = \sum_{n_0=0}^L \frac{\sum_L (y(\nu_\beta[n - n_0]))^2}{\sum_L (\nu_\beta[n - n_0])^2} \quad (24)$$

where $y(\nu_\beta)$ is the output of any NN explained in Sections 3 and 4 as a function of the input ν_β and $\sum_L (y(\nu_\beta[n - n_0]))^2$ the sum of their squared components.

In this way, given a noise type (for instance $\beta = 0$, that is white noise), if is unaltered after the filtering stage (because, for instance, the output of the NN is always equal to its input), $N_\beta = 1$. When the filter increases the noise level, $N_\beta > 1$ and when the filters reduces it, $N_\beta < 1$. Obviously, this is the only case in which NNs are useful.

Thus, if we set $\beta = 0$ (white noise), Eq. (24) yields

$$N_0 = \sum_{n_0=0}^L \frac{\sum_L (y(\delta[n - n_0]))^2}{L} \quad (25)$$

Making tests using Eq. (24) we get that these noise ratios are proportional to $L^{-\beta}$ when L tends to infinity.

In Figure 9, noise ratio vs. noise type for RBMs, DAEs, AEs are depicted for different values of H . For comparison purposes the results obtained with SVD filtering are also shown. Note that for noise types with $\beta < 0$, the noise ratio is increased with L , which in practice these noise types are the ones that affect the filters the most.

In all four cases, the noise that most affects them, like most filters, is noise with $\beta = -1$ (that is red noise). Besides, in all four cases, the lower the value of H , the greater the noise immunity the filter has. The lines of the NNs are more irregular than those of the SVD because they depend on the level of convergence during the training. Finally, we can see that the best results (almost two orders of magnitude with respect to SVD filtering) are those obtained with RBM. A better result is also observed in the upper limits of DAE than in those of AE because the former train is carried out with noise-free pulses. Finally, the most common noise in particle detectors has a continuous range of $-1 < \beta < -0.25$ [31, 32], therefore, these NNs are suitable to filter this type of noise.

6.3. Effect of the type of noise during the training

Figure 10 shows the error Δ_S for PSA when the NNs and the SVD filter are trained and tested with different types of noise (brownian, pink and white). We can observe that, as H increases, the error increases, in accordance with what was observed in Figure 6. In addition, as the noise becomes brownian (i.e. β decreases) in both learning and tests, a higher error is obtained but it can be mitigated lowering τ_{us} according to (20). Furthermore, as explained at the beginning of this section, a filter for brownian noise can be placed before the NN. Finally, DAE is the one that offers better results against any noise type because

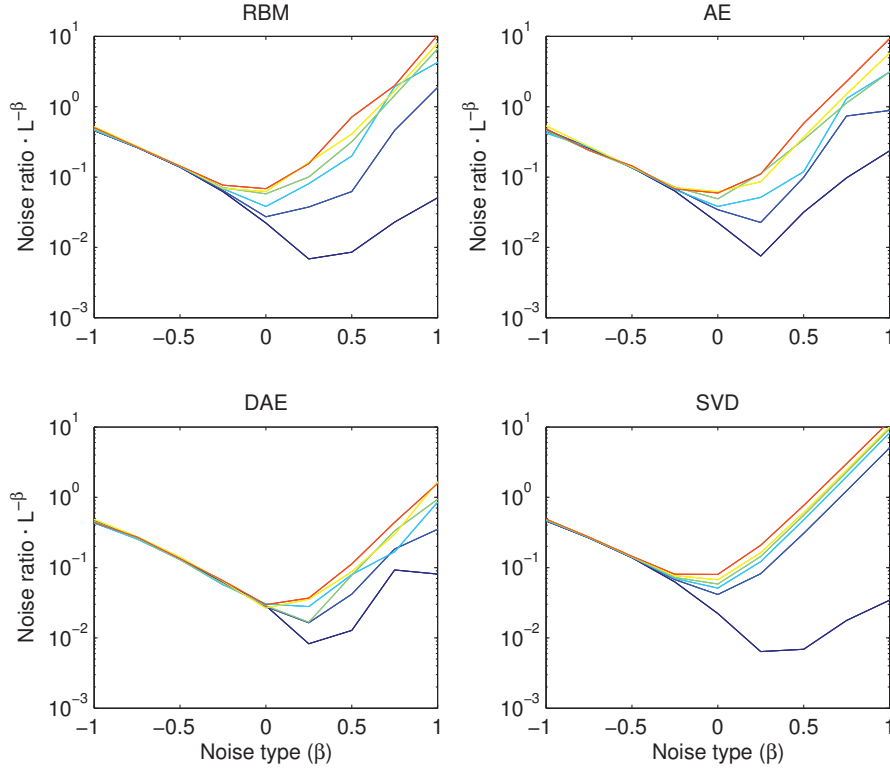


Figure 9: Noise ratio vs. β for RBM, DAE, AE and SVD.

266 it has the advantage that it is trained with pulses without noise, which also makes it have a more constant
 267 error. These conclusions are confirmed when we perform a PHA (Figure 11).

268 6.4. Results with pulses from a neutron monitor

269 Finally, a test in a real environment to check the proposed filtering has been performed. The main
 270 objective of this test has been to obtain similar results to those obtained in the experiments done with SVD
 271 filtering [9].

272 This test was performed in the Castilla-La Mancha Neutron Monitor (CaLMan) located in Guadalajara,
 273 Spain. This instrument consists of 15 proportional gas counter tubes. More information about features,
 274 setup and results of this instrument can be found in [33]. In both the cited experiment and the present test,
 275 an LND206 tube connected to a Canberra ACHNA98 preamplifier was used.

276 The raw data fed out from the preamplifier was digitized using a Data Acquisition system (DAQ) working
 277 at $L/\tau_s = 50$ Msamples/s and storing it in a PC. Pulses were stored in a text file that can be used multiple
 278 times without recapturing new data. In addition, it ensures that possible changes in the obtained results
 279 during the test are exclusively due to digital pulse processing. The total raw data length was of 45000 pulses

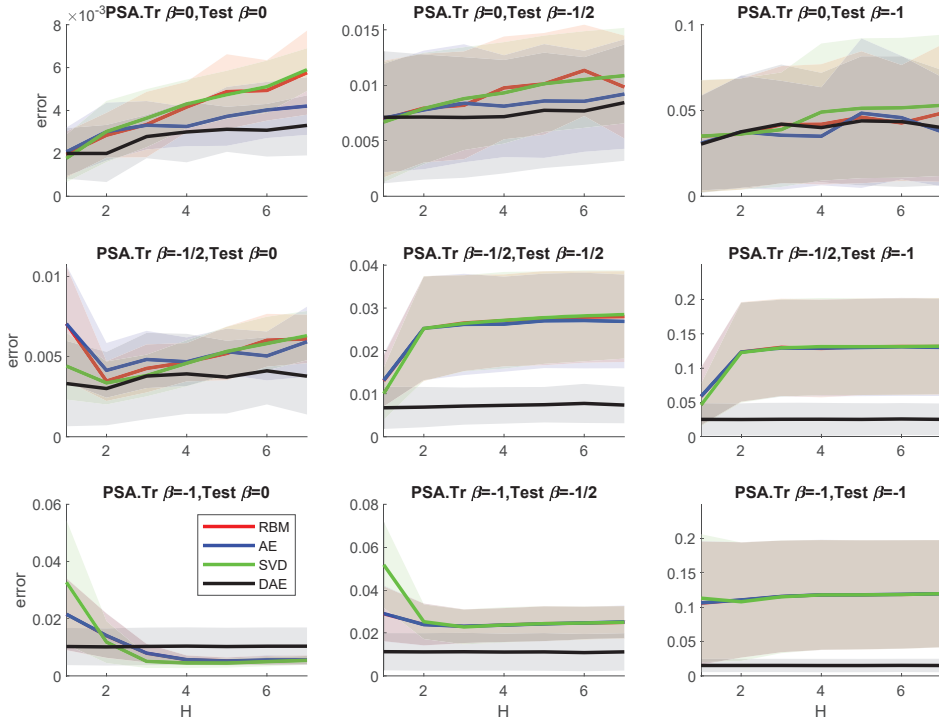


Figure 10: Mean and typical deviation of Δ_S (PSA) vs. training noise type. A set of 1000 samples were used to calculate them.

280 $\times 1002$ samples per pulse (i.e. $L = 1002$) captured during over 5 hours. To separate the input pulses, a
 281 trigger threshold of 1 V without any previous digital filtering was used.

282 To carry out a supervised learning with DAE, a pulse without noise obtained from simulations has been
 283 used during the training stage. Both in supervised and unsupervised learning, the number of epochs to train
 284 the NNs were 120000 with a learning rate $\alpha = 0.1$. These figures were selected taking into account that
 285 the more noise the training signal has, the more pulses and epochs are required to reduce the noise. It also
 286 was taken into account that preamplifier yields pulses of different shapes (Figure 12) making the training
 287 process harder. However, in all the tests the NNs converged.

288 Figure 13 shows a single pulse filtered with AE, SVD and DAE. For comparison purposes with different
 289 H values, SVD filtering was also depicted. We can observe that the shape of the DAE is adjusted to the
 290 pulse shape used during the training stage whereas the shape of \mathbf{y} is adjusted in proportion to H . Anyway,
 291 the pulse height is conserved independently of the pulse shape.

292 In Figure 14, the learning rate for the different NNs used and different H values ARE shown. We can
 293 observe that RBM and AE learning rates are similar and converge quicker than RBM.

294 Figure 15 depicts a series of histograms of the height of each pulse obtained during the experiment and
 295 filtered with NN. The histograms corresponding to SVD filtering were also added for comparison purposes.

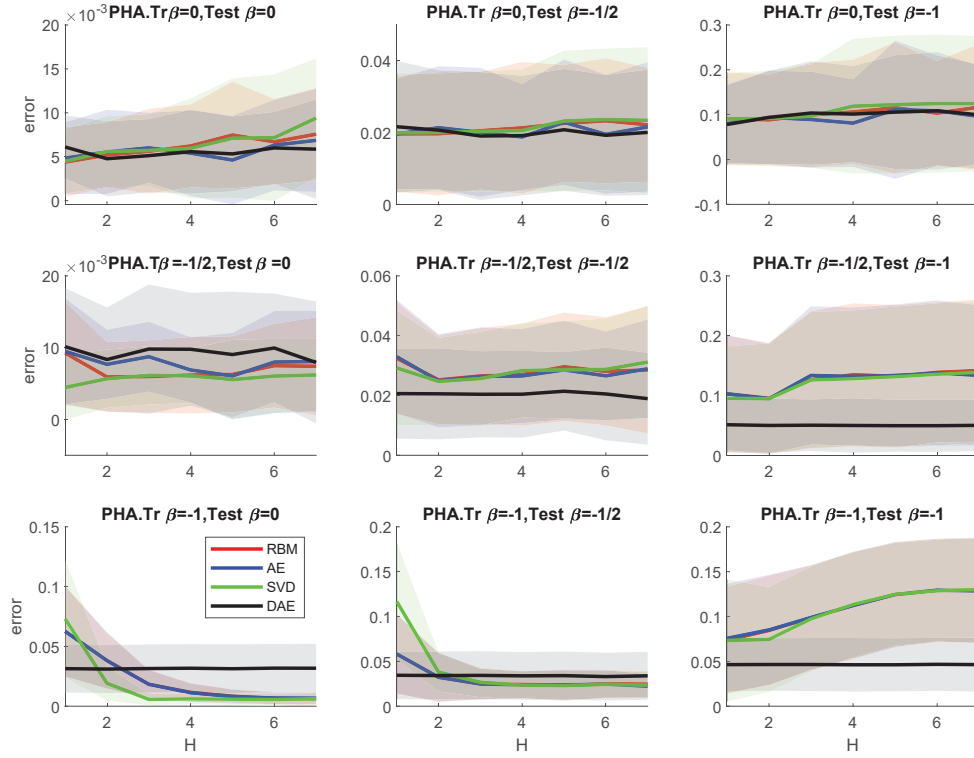


Figure 11: Mean and typical deviation of Δ_H (PHA) vs. training noise type. A set of 1000 samples were used to calculate them.

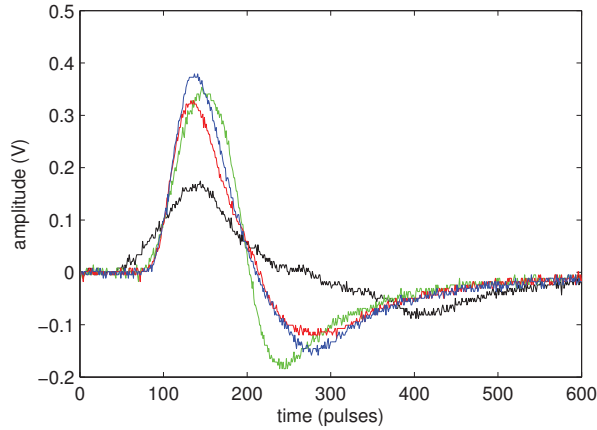


Figure 12: Pulses from the neutron monitor without filtering.

296 The number of channels used were 100. Since Eq. (18, 17) cannot be used to evaluate the obtained results,
 297 because the value of \mathbf{x}^* is not known, the filter quality was measured using the Full Width at Half Maximum
 298 (FWHM) which is defined as the width of the distribution at a level that is just half the maximum value of
 299 the peak divided by the location of the peak maximum [1].

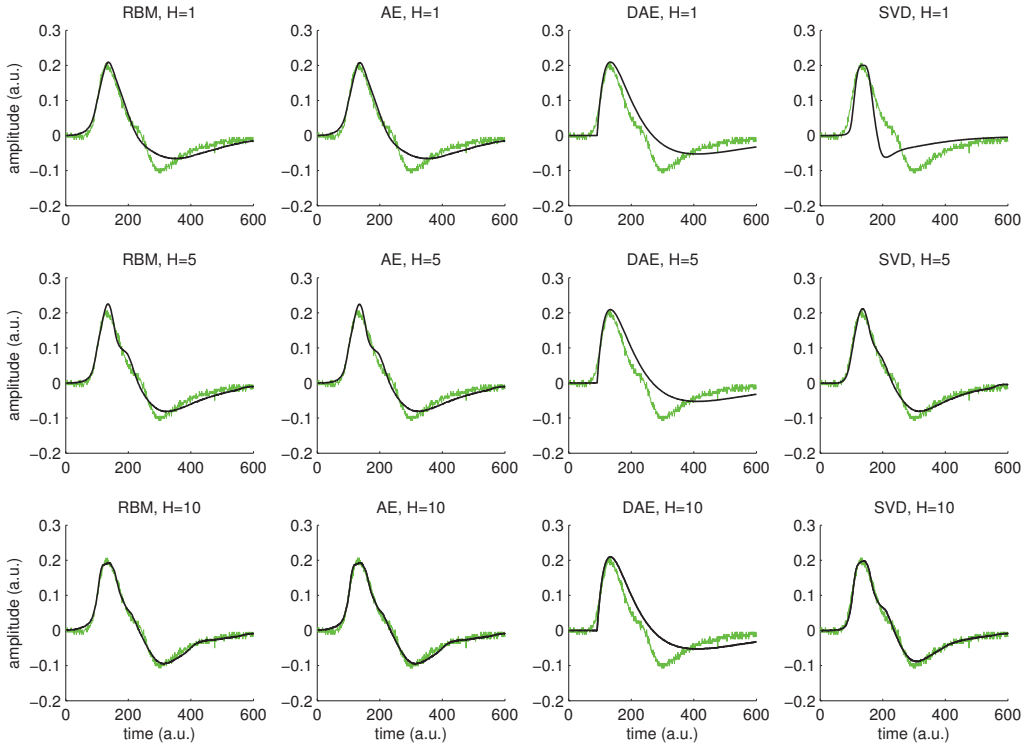


Figure 13: Filtering pulse comparison. The green line is the input pulse \mathbf{x} and the black solid line is the obtained output \mathbf{y} .

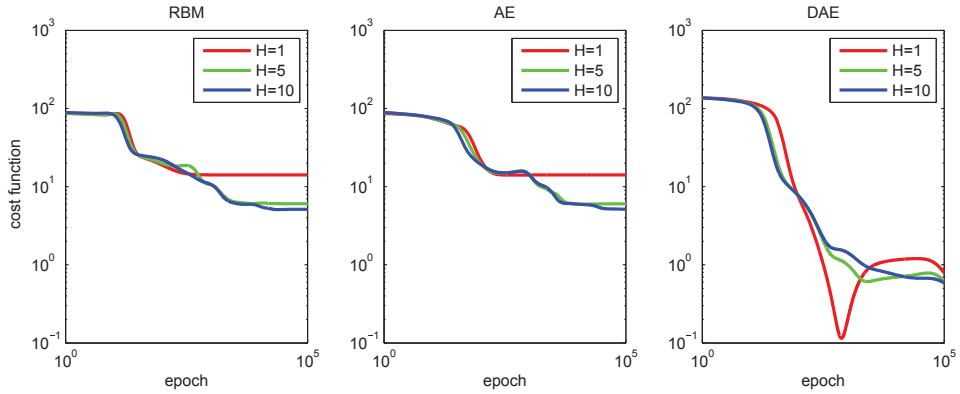


Figure 14: Cost function vs. epoch for the RBM, AE and DAE used in the neutron monitor with different H .

300 In all of histograms presented, the maximum was in channel 66 or 67 depending of the NN and H . The
 301 best results obtained were with the DAE followed by AE and RBM with high H . On the other hand, due to
 302 pulse shape irregularities RBM need a high H to learn all the pulse possible shapes and get a low FWHM.
 303 To finalize, we would like to point out from observations that these filtering methods do not handle

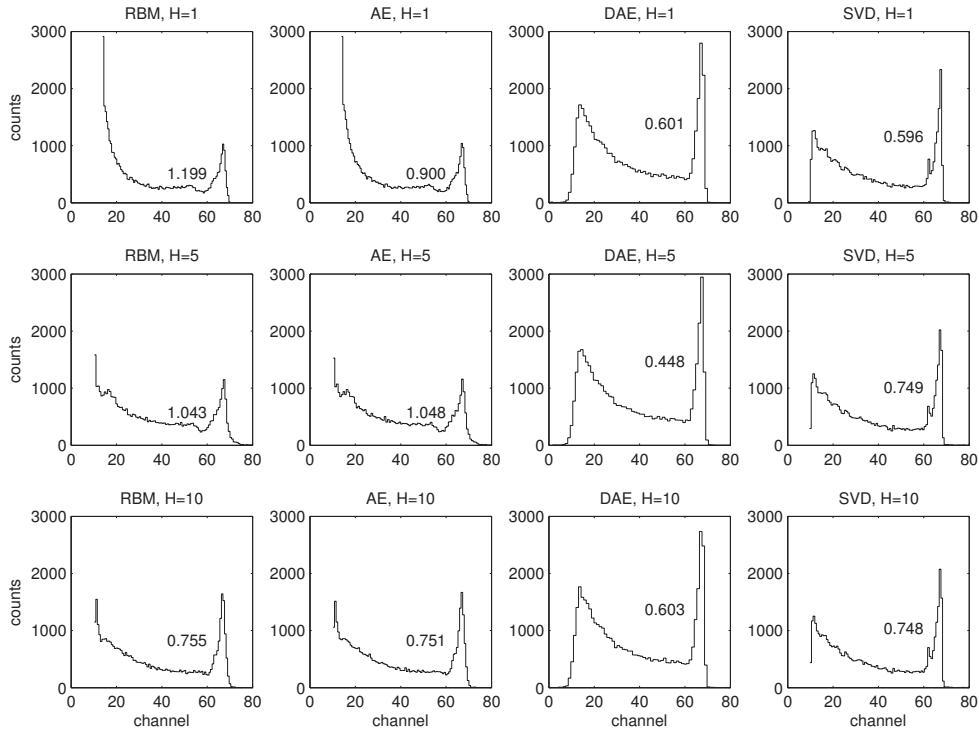


Figure 15: Histograms obtained using different filtering methods and different H with their corresponding FWHM.

304 properly with pile-up pulses. This fact also happened in previous works such as [8] and [9].

305 7. Conclusions

306 A novel filtering technique of isolated pulses based on NNs (namely RBMs, AEs and DAEs) has been
 307 presented in this article. These NNs have been compared with SVD in simulation and test with pulses
 308 coming from a neutron detector to evaluate its performance. In the same way that SVD, NNs have similar
 309 responses to noise and are less susceptible to white noise, but more to brownian noise than common digital
 310 filters such as FIR and IIR. In any case, the noise impact continues being dependent on the length of the
 311 pulse in the same way that common filters. However, despite the computation time for the training, AEs
 312 and DAEs improve significantly the results of SVD filtering. During the training stage, in all the tests the
 313 NNs converged without problems. Among these NNs, the best results were obtained using the DAE, despite
 314 pulses without noise are mandatory to train it. These methods based on linear NNs overcome the results
 315 obtained with common digital filters and are relatively easy to implement in software and therefore suitable
 316 for analysis of pulses coming from particle detectors.