Understanding expressive intonation: Casals' Bach beyond conscience

Comprendiendo la entonación expresiva: el Bach de Casals más allá de lo consciente

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Abstract

Research on intonation has mainly sought for classifying and/or expressive explanations for performers' strategies. In the field of music psychology and music perception, such explanations have been explored in terms of interval direction, size, or type; in the field of performance analysis, to which this article belongs, investigation on intonation has been not only scarce but also limited to short excerpts. In this context, this article explores Pau Casals' intonational practice specific to his recording of Bach's E flat major prelude for solo cello. To do so, on the basis of exact empirical measurements, it places such practice alongside the cellist's conscious, theoretical recommendations apropos what he called "expressive" string intonation, showing that the interpretation of the latter should is not straightforward. It also proposes several reference points and tuning systems which could serve as models for Casals' practice and looks for explanations *beyond* simple interval classification. In this manner, it ultimately proposes a structural function for intonation, in partnership with tempo and dynamics. Similarly, it understands Casals' intonational practice not as a choice between but as a *compromise for multiple options* in tuning systems (mostly equal temperament and Pythagorean tuning), reference points (the fundamental note of the chord and the immediately preceding tone), the nature of the compositional materials (harmonic and melodic), and, most importantly, structure and expression.

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Key words: intonation; expression; structure; Casals; Bach cello suites; performance analysis; tempo; dynamics; equal temperament; Pythagorean tuning.

Resumen

La investigación acerca de la afinación ha pretendido explicar las decisiones de los intérpretes en base a clasificaciones interválicas o a motivos expresivos. En el campo de la psicología y la percepción de la música, este tipo de explicaciones han estado relacionadas con la dirección, el tamaño o el tipo de intervalo; en el campo del análisis empírico de la interpretación, en el que este trabajo se inserta, el estudio de la afinación no solo ha sido escaso sino que se ha limitado al análisis de breves fragmentos. En este contexto, este artículo explora la práctica afinatoria de Pau Casals en su grabación del preludio en mi bemol mayor para violonchelo solo de Bach. Para ello, basándose en mediciones empíricas exactas, sitúa a dicha práctica junto a las conscientes recomendaciones teóricas del chelista acerca de lo que él llamaba «afinación expresiva» en la cuerda, demostrando que el significado de dicho término no es tan sencillo de interpretar como podría parecer. También propone diversos puntos de referencia y sistemas de afinación como posibles modelos para Casals y busca explicaciones *más allá* de una simple clasificación de los intervalos. De esta manera, en última instancia este trabajo propone que la afinación puede desempeñar una función estructural junto al *tempo* y a las dinámicas. Del mismo modo, entiende la práctica afinatoria de Casals no como una elección entre sino como una *unión entre múltiples opciones* para el sistema de afinación (mayoritariamente temperamento igual y afinación pitagórica), el punto de referencia (la nota fundamental del acorde y la nota inmediatamente precedente), la naturaleza de los materiales compositivos (armónica y melódica), y, sobre todo, una unión de estructura y expresión.

Palabras clave: afinación; expresión; estructura; Casals; suites para cello solo de Bach; análisis de la interpretación; *tempo*; dinámicas; temperamento igual; afinación pitagórica.

Casals owned his Bach, for he felt really owned by Bach[1].

I. Casals and the sense of "expressive intonation"

With that assertion, in 1985 Richard Taruskin declared his respect and admiration for Pau Casals' (1876-1973) performance style in playing Johann Sebastian Bach's suites for solo cello, as well as his sonatas for viola da gamba and harpsichord (on the cello, of course). Certainly, the cellist did not adhere to the "historically informed" trends that would later become widespread and which

[1] Richard Taruskin, "Throwback or harbinger?", in *Text and Act—Essays on Music and Performance* (Oxford and New York: Oxford University Press, 1985), 302.

would potentially reach the spheres not only of articulation, phrasing, and absolute pitch[2], but also of intonation[3]. To Casals, intonation was a "question of conscience", thinking always in terms of a chain in which each note "is like a link […]—important in itself and also as a connection between what has been and what will be", and in which "each interval has a "specific sense of belonging and/or direction". In fact, he specifically advocated against the "fixed and equidistant semitones" of the equal temperament, preferring instead smaller diatonic semitone steps, especially those leading towards the first, the fourth, and the fifth degrees of the scale, and even more so in fast compositions. This was to him the "expressive intonation" to which any performer must aspire, a "dynamic process expressing the organic relationship of the notes in a musical context"[4].

Musical context, harmonic attraction towards the most important degrees in the scale—those that, furthermore, are a fifth apart the tonic note—, small diatonic semitones, and linear note-to-note relationships. These theoretically typify a system close to the Pythagorean tuning that some scholars discerned in the practices of some performers contemporary to Casals[5]. However, more recent research has not only shown that that early scholarship was based on inaccurate methods and, hence, data[6]: it has also demonstrated that string players rarely adjust their notes to the perfect-fifths Pythagorean proportions[7]. Interestingly, other accounts of Casals' recommendations for performance stress that,

[2] In a comprehensive study of recordings of Bach's Brandenburg concertos, Fabian places an emphasis on articulation and tuning as definers of historical performance; see Dorottya Fabian, *Bach Performance Practice, 1945-1975. A Comprehensive Review of Sound Recordings and Literature* (New York: Routledge, 2017). In a previous analysis of recordings of Bach's partitas and sonatas for solo violin, she also speaks of phrasing as "the feature that occupies the ultimate seat of interactions" in performance; see Dorottya Fabian, *A Musicology of Performance: Theory and Method Based on Bach's Solos for Violin* (Cambridge: Open Book Publishers, 2015). In a similar vein, Ornoy considers pitch, rhythmic interpretation, and ornamentation as elements potentially differentiating historically informed from mainstream performances of Bach's music; see Eitan Ornoy, "Between theory and practice: Comparative study of early music performances", *Early Music* 32, no. 2 (2006): 233-247.

[3] Previous studies have nonetheless preliminary discarded the hypothesis that intonation may be a distinctive element of "historically informed" performance practice. See, for instance, Eitan Ornoy, "An empirical study of intonation in performances of J.S. Bach's sarabandes: Temperament, 'melodic charge', and 'melodic intonation'", *Orbis Musicae* 1 (2007): 37-76 and "Between theory and practice…".

[4] David Blum, *Casals and the Art of Interpretation* (London: Heinemann, 1977), 101-103.

[5] For instance, Paul C. Greene, "Violin performance with reference to tempered, natural, and Pythagorean intonation", in *Studies in the Psychology of Music*, ed. Carl E. Seashore (Iowa City: University of Iowa, 1936), 249; and James F. Nickerson, "Intonation of solo and ensemble performances of the same melody", *The Journal of the Acoustical Society of America* 21, no. 6 (1949): 593-595.

[6] Cornelia Yarbrough and Dana L. Ballard, "The effect of accidentals, scale degrees, direction, and performer opinions on intonation", *Update: Applications of Research in Music* 8, no. 2 (1990): 19-22.

[7] See, for instance, Rebekah Ann Brown, "Dynamics of intonation in performances by artist violinists" (doctoral thesis, Indiana University, 1996), 101, https://search-proquest-com.universidadviu.idm.oclc.org/docview/304301317; John M. Geringer, "Intonational performance and perception of ascending scales", *Journal of Research in Music Education* 26, no. 1 (1978): 38; and Yarbrough and Ballard, "The effect of accidentals…", 21-22.

even to him, there were exceptions to the general rule of the "expressive intonation", these being the double stops. In them, the player must seek for a "compromise between expressive intonation and the 'tempered' one"[8]. In this "compromise", diatonic semitones are not as small as in the Pythagorean tuning and at the same time not as large as in the equal temperament. It appears, then, that Casals preferred to play close to the just intonation in the passages with double stops, the same as violinists of Bach's time, such as Prelleur and Geminiani, advocated in both melodic and harmonic contexts[9]. However, Casals' "non-expressive intonation", i.e., the normative one against which to define the "expressive" one, "could well have been E[qual] T[emperament], since his semitone steps [in his recording of the Sarabande from Bach's Suite No. 5 in c minor, BWV 1011] are always narrower, though not always excessively narrower, than the 100 cents of E[qual] T[emperament]"[10]. Therefore, in Casals' case we seem to be facing a fundamental contradiction between the theory and the practice, or, perhaps, between the theory and a practice that was not as inflexible as the cellist's comments may be interpreted to indicate.

As a matter of fact, as a cello student I remember being instructed to listen to the natural resonance of the strings when playing double stops, and to the piano when playing a duo sonata, for instance. I also remember being—discordantly—told to slightly narrow the fifths between the open strings of my instrument, not only when playing with an equally-tempered piano, but also when playing in a string quartet or even a solo piece. That is, as string performers we are accustomed to constantly adjust to the directional tension of the leading notes (Pythagorean tuning)[11], the natural resonance of the instrument produced by the harmonic series (just intonation)[12], and the equidistant system of the equal temperament[13], even if we are not consciously aware of the theoretical ratios between the notes in the three systems[14].

[8] J. Ma. Corredor, *Conversations with Casals* (London: Hutchinson, 1956), 197.

[9] David D. Boyden, "Prelleur, Geminiani, and just intonation", *Journal of the American Musicological Society* 4, no. 3 (1951): 202-219. Even as late as in 1869 Cornu and Mercadier, who also studied solo violin performance, defended that just intonation was preferred in harmonic contexts; see Alfred Cornu and Ernest Mercadier, "Sur les intervalles musicaux", *Comptes rendus habdomadaires des séances de l'académie des ciences* 58 (1869): 301-308.

[10] Peter Johnson, "'Expressive intonation' in string performance: Problems of analysis and interpretation", in *The Music Practitioner: Research for the Music Performer*, ed. Jane W. Davidson (Aldershot: Ashgate, 2004), 83.

[11] The Pythagorean tuning is based on the perfection of all fifths, thus producing very small diatonic semitones and two types of major seconds. Also major thirds are larger than in the just intonation and the equal temperament. For a detailed chart of the interval sizes in the three tuning systems discussed in this article, see table 1 below.

[12] The just intonation is based on the interval ratios naturally produced by the harmonic series of any complex sound. In it, major thirds are smaller than in the equal temperament and the Pythagorean tuning. To achieve them, it theoretically shrinks one every four fifths in the circle. See table 1 for proportions and figure 1 for the harmonic series.

[13] The equal temperament is based in the equal size of all semitones. It gained force in the nineteenth century due to the need for a system allowing for diatonic as well as chromatic modulation. For a detailed explanation of the various tuning systems, see Jutta Stüber, *Die Intonation des Geigers* (Bonn: Verlag für systematische Musikwissenschaft GmbH, 1989); and Leon Gunther, "Tuning, intonation, and temperament: choosing frequencies for musical notes", in *The Physics of Music and Color* (New York: Springer, 2012), 353-381.

[14] Some literature has defended that the mesotonic system, or the system in which all fifths are equally

Perception may vary depending on the individual[15], yet most would agree that Casals' rendition of Bach's cello suites sounds in tune. If Pythagorean tuning is not always rated high[16], does this mean that Casals did not say the truth when speaking of his intonation strategies? Did his "expressive" principles apply to fast melodic contexts only or also to more harmonic, vertical passages? In other words, did the potentially just "compromise" which he advocated for double stops apply to other kinds of harmonic writing, such as arpeggios? And even to more melodic passages, as it seems was the practice in Bach's times?

To explore these issues, in the following pages I will analyse Casals' recording of the prelude from Bach's Suite no. 4 for solo cello, BWV 1010, in E flat major[17]. The piece is particularly suitable for the proposed analysis, as, differently from the c minor Sarabande that has already been studied as performed by Casals[18], its key makes it almost impossible for performers to exploit the open strings of the cello. They can only do so for the third of the tonic chord in the second octave (G_2)[19], and also for the tonic of the relative key, which is significantly emphasised on the score throughout the piece. That means that cellists barely depend on their initial decisions when tuning the instrument or on the natural fluctuations of the strings, especially if these are made of gut. Rather, tuning relies almost exclusively on the precision of their left-hand fingers[20]. Furthermore, the tripartite structure

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narrowed by a comma to attain the major thirds of the harmonic series, was the standard in keyboard instruments of Bach's time, yet very few authors suggested it for the violin or other fretted string instruments. For an account of historical violin intonation, see Patrizio Barbieri and Sandra Mangsen, "Violin intonation: A historical survey", *Early Music* 19, no. 1 (1991): 69-88. For a discussion of mesotonic, or mean-tone, intonation, see esp. Ornoy, "An empirical study of intonation…".

[15] For a literature review on the topic, see Robert A. Duke, John M. Geringer and Clifford K. Madsen, "Effect of tempo on pitch perception", *Journal of Research in Music Education* 36, no. 2 (1988): 109-110. Later research has confirmed such findings.

[16] In fact, although Ward and Martin defend that equal temperament is usually preferred at least in harmonic contexts. Loosen has found that preferences greatly vary depending on training, as pianists tend to favour equal temperament whereas violinists normally rank excerpts played to Pythagorean tuning higher. In any case, in this experiment there were significant divergences between the individuals in each group. See Franz Loosen, "The effect of musical experience on the conception of accurate tuning", *Music Perception* 12, no. 3 (1995): 291; and W. D. Ward and D. W. Martin, "Psychophysical comparison of just tuning and equal temperament in sequences of interval tones", *The Journal of the Acoustical Society of America* 33, no. 5 (1961): 588.

[17] Original recording: *THE BACH* SOCIETY, vol. 8, EMI His Master's Voice D.B.6541, 1948-1950 (recorded Paris 1739), 78 rpm. Remastered in *J.-S. Bach. 6 Suiten für Violoncello / Les 6 Suites pour violoncelle / The 6 Cello Suites. PABLO CASALS*, EMI, CHS 7 61027 2, CD 2 (1988). The recording is available online at https://www.youtube.com/watch?v=LivsP2dUce0.

[18] Johnson, "'Expressive intonation' in string performance…".

[19] Octave indices follow international standard, the central octave, i.e. that which contains the A at ~440 Hz, being octave no. 4.

[20] Although Lewis and Cowan deny any influence of intensity—or dynamics—on intonation on the notes played on open string, authors such as Fletcher, Haynes, and Sogin set forth that louder notes tend to be played sharper. Similarly, Dorottya Fabian distinguished the same effect in Szigeti's recording of Mendelssohn's violin concerto. See,

of the piece sets the harmonic, arpeggio writing of the opening and closing sections (bars 1-49 and 85-91) against the more linear quality of the quasi-improvised central part (bars 49-84)[21]. In this manner, the assessment of Casals' intonation strategies in both harmonic and melodic contexts can be established within a single performance event, thus ruling out the potential influence of different recording conditions, periods in his playing style, instruments, acoustics, repertoires, or even compositional writing.

Before discussing the results of the analysis, I will place Casals' strategies in a wider theoretical context in order to discern whether some of his traits were unique to him. I will also explain the empirical methods employed in the analysis. Finally, I will combine intonation with tempo and dynamics data to offer an explanation for Casals' tuning in Bach's prelude. Even if he was not "conscious" of those effects and, as a consequence, did not address them in his conversations on music making, the analysis supports that intonation can be not only "expressive" but also "structural"—if a separation between them can indeed be drawn[22].

II. Performed intonation

Being intonation such an integral part of string playing, it is surprising that very few works in the field of performance analysis have been devoted to its investigation. Significantly, two recent publications by Dorottya Fabian, both focused on Bach's performance practice, mention it just in passing, quoting contemporary critics' appraisals on the performers' "accurate" or "good" intonation[23]. Similarly, most previous research has primarily explored the effects of musicians' handling of tempo and dynamics from various angles, including expressive, structural, and stylistic

by alphabetical order, Dorottya Fabian, "The recordings of Joachim, Ysaÿe and Sarasate in the light of their reception by nineteenth-century British critics", *International Review of the Aesthetics and Sociology of Music* 37, no. 2 (2006): 196; H. Fletcher, "Loudness, pitch and the timbre of musical tones and relation to intensity, the frequency, and the overtone structure", *Journal Acoustical Society of America* 6 (1934): 59-69; Bruce Haynes, "Beyond temperament: non-keyboard intonation in the 17th and 18th centuries", *Early Music* 19, no. 3 (1991): 356-381; Don Lewis and Milton Cowan, "The influence of intensity on the pitch of violin and 'cello tones'", *The Journal of the Acoustical Society of America* 8 (1936): 20-22; and David W. Sogin, "An analysis of string instrumentalists' performed intonational adjustments within an ascending and descending pitch set", *Journal of Research in Music Education* 37, no. 2 (1989): 104-111.

[21] An annotated score of the Prelude is provided in the Appendix to this article. The edition is based on the manuscript D-B Mus. Ms. Bach P 269, consulted in the facsimile reproduction of Bärenreiter-Verlag (Kassel: BA 320, source A).

[22] For an exploration of the structural import of other "expressive" effects in Casals, such as the asynchronisation between the cello and the piano in Casals and Horszowski's recording of the second movement of Brahms's cello sonata in F major, see Ana Llorens, "Recorded asynchronies, structural dialogues: Brahms's *Adagio affettuoso*, op. 99ii, in the hands of Casals and Horszowski", *Music Performance Research* 8 (2017): 1-31.

[23] A contrasting example of this phenomenon occurs in Fabian' article on Joachim, Ysaÿe, and Sarasate's recordings, where claims on the violinists' intonational practices are grounded on empirical data; see Fabian, "The recordings of Joachim, Ysaÿe and Sarasate…": 196-197.

considerations[24]. Undoubtedly, this lack of analytical research into intonation is partly due to the methodological complications, as the extraction of the data from commercial recordings is still a mostly manual, highly time-consuming process[25]. As a corollary, some scholars analysing commercial recordings by professional artists[26], notably of Bach's unaccompanied music, have only focused on specific passages or a few intervals for analysis. Among them, Peter Johnson justifies his selecting just a few passages from Bach's c minor cello sarabande and Beethoven's string quartets opp. 132iii and 135iii[27] on the grounds that "to analyse intonation we have to work from small samples, and these need to be representative or symptomatic of the larger musical context"[28]. Yet, if "intonation is inconsistent throughout the course of a musical work"[29], how can one be sure that the select passages are indeed representative of the performance as a whole? To determine which passages are indeed representative of the performance, small-scale data should be placed in dialogue with large-scale, statistical results, as I try to do here.

With a focus on the latter perspective, i.e., on the large-scale evaluation of the data, research on intonation in string performance—and on intonation in general—has mostly belonged to the fields of music psychology and music education—note the journal titles for most of the literature discussed in the subsequent paragraphs. It is important to note that virtually all this research was performed under artificial conditions, i.e., with musicians either playing isolated intervals or, at the

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[24] The richness of the literature on these topics greatly surpasses the scope of this article. To mention just a few fundamental contributions, see Nicholas Cook, *Beyond the Score: Music as Performance* (New York: Oxford University Press, 2013); Nicholas Cook, "Performance analysis and Chopin's mazurkas", *Musicae Scientiae* 11, no. 2 (2007): 183-207; Daniel Leech-Wilkinson, *The Changing Sound of Music: Approaches to Studying Recorded Musical Performance* (London: Centre for the History and Analysis of Recorded Music, 2009); John Rink, ed., *Musical Performance: A Guide to Understanding* (Cambridge: Cambridge University Press, 2002); and John Rink, *The Practice of Performance: Studies in Musical Interpretation* (Cambridge: Cambridge University Press, 1995).

[25] Some attempts have been made at automatically extracting intonation data from MIDI files; see Johanna Devaney and Daniel P. W. Ellis, "An empirical approach to studying intonation tendencies in polyphonic vocal performances", *Journal of Interdisciplinary Music Studies* 2, nos. 1-2 (2008): 141-156.

[26] For instance, Brown, "Dynamics of intonation…"; John M. Geringer, "Eight artist-level violinists performing unaccompanied Bach: Are there consistent tuning patterns?", *String Research Journal* 8 (2018): 51-61; Arnold Milroy Small, "An objective analysis of artistic violin performance", in *University of Iowa Studies in the Psychology of Music*, vol. 4, ed. Carl E. Seashore (Iowa City: University of Iowa, 1937), 172-231; Ornoy. "An empirical study of intonation…"; and Ornoy, "Between theory and practice". Ornoy is the exception, as he analyses entire recordings.

[27] Johnson, "'Expressive intonation' in string performance…"; and Peter Johnson, "Intonation and interpretation in string quartet performance: the case of the flat leading note", in *6th International Conference of Music Psychology, Keele, August 2000*, accessed June 27, 2014, http://www.escom.org/proceedings/ICMPC2000/Tue/Johnson.htm.

[28] *Ibid*., §1.

[29] Robert A. Duke, "Wind instrumentalists' intonational performance of selected musical intervals", *Journal of Research in Music Education* 33, no. 2 (1985): 102.

most, scales[30], or being asked to tune and/or judge prerecorded excerpts[31]. In other experiments, performers were asked to play longer excerpts from well-known works, such as Haydn's *Emperor* quartet or Bach/Gounod's *Ave Maria*, yet the studies were conducted in laboratories and on short passages too[32].

Scholarship tends to agree on the fact that string instrumentalists prefer one tuning system over the other depending on both training[33] and musical context[34], varying not only across pieces but even "from note to note"[35]. Already Cornu and Mercadier stated that, in the middle of the nineteenth century, violinists allegedly opted for the just intonation in harmonic passages and for Pythagorean intervals in melodic ones[36]. Along similar lines, Barbieri and Mangsen have demonstrated that, at

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[30] For instance, Frank A. Edmonson, "Effect of interval direction on pitch acuity in solo vocal performance", *Journal of Research in Music Education* 20, no. 2 (1972): 246-254; Geringer, "Intonational performance…"; Vincent J. Kantorski, "String instrument intonation in upper and lower registers: The effects of accompaniment", *Journal of Research in Music Education* 34, no. 3 (1986): 200-210; Franz Loosen, "Intonation of solo violin performance with reference to equally tempered, Pythagorean, and just intonations", *Journal of the Acoustical Society of America* 93, no. 1 (1993): 525-539; Clifford K. Madsen, "The effect of scale direction on pitch acuity in solo vocal performance", *Journal of Research in Music Education* 14, no. 4 (1966): 266-275; James A. Mason, "Comparison of solo and ensemble performances with reference to Pythagorean, just, and equi-tempered intonations", *Journal of Research in Music Education* 8, no. 1 (1960): 31-38; Andrzej Rakowski, "Intonation variants of musical intervals in isolation and in musical contexts", *Psychology of Music* 18 (1990): 60-72; Sogin, "An analysis of string instrumentalists' performed intonational adjustments …"; and Yarbrough and Balland, "The effect of accidentals…".

[31] For instance, in J. Elliot, J. R. Platt and R. J. Racine, "Adjustment of successive and simultaneous intervals by musically experienced and inexperienced subjects", *Perception & Psychophysics* 42, no. 6 (1987): 594-598; John M. Geringer, "Tuning preferences in recorded orchestral music", *Journal of Research in Music Education* 24, no. 4 (1976): 169-179; John. M. Geringer and Anne C. Witt, "An investigation of tuning performance and perception of string instrumentalists", *Bulletin of the Council for Research in Music Education* 85 (1985): 90-101; Franz Loosen, "Tuning of diatonic scales by violinists, pianists, and nonmusicians", *Perception & Psychophysics* 56, no. 2 (1994): 221-226; Franz Loosen, "The effect of musical experience…"; and Rudolf A. Rasch, "Perception of melodic and harmonic intonation of two-part musical fragments", *Music Perception* 2, no. 4 (1985): 441-458.

[32] See Barry Rex Garman, "The effects of accompaniment texture and contextual pitch distance on string instrumentalists' intonational performance" (doctoral thesis, University of Miami, 1992), https://search-proquest-com.universidadviu.idm.oclc.org/docview/303969349; John M. Geringer, Rebecca B. MacLeod, and Justine K. Sasanfar, "In tune or out of tune: Are different instruments and voice heard differently?", *Journal of Research in Music Education* 63, no. 1 (2015): 89-101; Nickerson, "Intonation of solo and ensemble performances…"; Charles Shackford, "Some aspects of perception I: Sizes of harmonic intervals in performance", *Journal of Music Theory* 5, no. 2 (1961): 162-202; and Charles Shackford, "Some aspects of perception II: Interval sizes and tonal dynamics in performance", *Journal of Music Theory* 6, no. 1 (1962): 66-90.

[33] See note 16 above.

[34] On the one hand, Mason defends that harmony exerts a more powerful influence on performers' intonation practices than melody, whereas, on the other, Nickerson states contrastingly. See Mason, "Comparison of solo and ensemble performances…"; and Nickerson, "Intonation of solo and ensemble performance…".

[35] Geringer, "Eight artist-level violinists…", 59.

[36] Cornu and Mercadier, "Sur les intervalles musicaux".

least until the middle of the eighteenth century, violinists "played in a kind of 'just' or mean-tone intonation. In fact[,] we have evidence that (1) their major 3rds were pure and (2) sharps were played lower than the enharmonically equivalent flats"[37]. As they tuned the open strings in pure fifths, they avoided them to elude Pythagorean intervals.

However, towards the middle of the eighteenth century, "sharps began to be tuned higher than the enharmonically equivalent flats: this habit was due to the new 'functional' and 'dynamic' role that semitones had in the modern harmonic-tonal system (e.g., the pull of the tonic on the leading note and of the sixth degree on the minor 7th made—respectively—the sharps to raise and the flats to lower)"[38]. It seems, then, that by Casals' time this kind of practice was still in vogue, probably in conjunction with a certain tendency to the equal temperament in some contexts, as we have already discussed. In fact, modern research has proved that this is still the most frequent situation in contemporary performance, with string players normally preferring the equal temperament and the Pythagorean tuning[39]. In a nutshell, the just intonation was preferred in the eighteenth and the early nineteenth centuries, with Pythagorean and equi-tempered tuning gaining prevalence afterwards. However, exceptions always occur, as is the case of the Lindsays quartet, who in their recording of Beethoven's op. 135iii aimed at just intonation, to a potentially expressive effect[40]. With respect to training, more experienced musicians purportedly tend to sharpness, although this has been challenged by some laboratory experiments[41].

In general, musicians under both performance and experiment conditions show a tendency towards intonational sharpness. Sharpness has been noticed not only in string playing, but also in woodwinds and the voice, in such a way that "intonation errors in the performance of music would

[37] Barbieri and Mangsen, "Violin intonation…", 85.

[38] *Ibid.*, 86-87.

[39] See, for instance, Geringer, "Eight artist-level violinists…", 57; Loosen, "Intonation of solo violin performance…", 525; and Loosen, "The effect of musical experience…", 291.

[40] Johnson, "Intonation and interpretation…", 4.

[41] Older subjects have shown opposite proclivities depending on the experiment, i.e. towards flatness (Duke, Geringer and Madsen, "Effect of tempo…", 116; Clifford K. Madsen, Frank A. Edmonson, and Charles H. Madsen, "Modulated frequency discrimination in relationship to age and musical training", *The Journal of the Acoustical Society of America* 46, no. 6B (1969): 1468-1472) or towards sharpness (Geringer, "Intonational performance…", 39; and Madsen, "The effect of scale direction…", 272).

be expected to consist primarily of errors in the sharp direction"[42]. This tends to be correlated with an alleged better flatness discrimination[43].

This seems to be the case of major seconds and, especially, major thirds[44], and even of octaves[45], which tend to be widened in performance. Specifically in Bach's performance practice, such enlarged major thirds may correspond to the composer's standard temperament system[46]. For their part, minor thirds and, most often, minor seconds are frequently narrowed. Whereas perfect fourths and perfect fifths in average show little deviation in performance from their theoretical values[47], these being very similar in the three tuning systems considered here (table 1), in some contexts descending fourths have resulted to be notably dissimilar among musicians[48]. In any case, more dissonant intervals, such as diminished sevenths, are normally performed with considerable deviation from their theoretical size[49]. Interestingly, one study ranked—descending—minor sixths as even more variable than any other intervals[50].

However, trying to find an explanation for such intonational deviations, scholarship has obtained highly contrasting results, with no consensus on the effects of interval direction, interval

[42] Geringer, "Tuning preferences…", 175. For concordant conclusions, see Garman, "The effects of accompaniment texture…", 86; Geringer, "Intonational performance and perception…", 36; John M. Geringer and David W. Sogin, "An analysis of musicians' intonational adjustments within the duration of selected tones", *Contributions to Music Education* 15 (1988): 4; Geringer and Witt, "An investigation of tuning performance…": 93; Madsen, "The effect of scale direction…", 271; Mason, "Comparison of solo and ensemble performances…", 35; Kantorski, "String instrument intonation…", 204 (although to him differences were not statistically significant); Rita S. Salzberg, "The effects of visual stimulus and instruction on intonation accuracy of string instrumentalists", *Psychology of Music* 8, no. 2 (1980): 47-48; Shackford, "Some aspects of perception I…", 185, with an emphasis on perfect fifths; and Yarbrough and Ballard, "The effect of accidentals…", 20. The only reviewed study in which such tendency towards sharpness was not found is Duke, "Wind instrumentalists' intonational performance…", 110, perhaps due to the instruments' different nature (recall that most of the others focused on string playing).

[43] See Duke, Geringer, and Madsen, "Effect of tempo…", 122; Geringer, "Intonational performance…", 32; Madsen, Edmonson, and Madsen, "Modulated frequency discrimination…", 1468; and Clifford K. Madsen and John M. Geringer, "Discrimination between tone quality and intonation in unaccompanied flute/oboe duet", *Journal of Research in Music Education* 29, no. 4 (1981): 312 (on woodwind playing).

[44] Brown, "Dynamics of intonation…", 106; Greene, "Violin performance with reference…", 241-242; Shackford, "Some aspects of perception I…", 189; Shackford, "Some aspects of perception II…", 69; and Small, "An objective analysis…", 200.

[45] Rakowski, "Intonation variants of musical intervals…", 67-68; on the contrary, Brown ("Dynamics of intonation…", 113-114) and Loosen ("Intonation of solo violin performance…", 537) found no such stretching.

[46] Ornoy, "Between theory and practice…", 236.

[47] Brown, "Dynamics of intonation…", 101; Greene, "Violin performance with reference…", 250; Rakowski, "Intonation variants of musical intervals…", 63; and Shackford, "Some aspects of perception II…", 69.

[48] Edmonson, "Effect of interval direction…", 250 (on vocal performance).

[49] Ornoy, "Between theory and practice…", 236.

[50] Edmonson, "Effect of interval direction…", 250 (on vocal performance).

size, register, or vibrato. It has been proposed that ascending intervals are alternatively enlarged[51] or contracted[52] in performance, and that, consequently, the deviation in descending intervals can also be in both directions. Ascending intervals have also been observed to be played both more[53] and less[54] accurately than descending patterns. Taking a step forward, Rakowski tried to offer a two-interval explanation, according to which large ascending intervals are "compensated in the following interval by a smaller value if the melody continues in the same direction, and by a larger value if the melody changes direction"[55]. Given such conflicting outcomes, some studies have determined that interval direction has no effect on intonational practice[56].

Along similar lines, small intervals have been reported as both stretched[57] and narrowed[58] in performance, and as more[59] and less[60] accurate than large ones. Even if, on occasions, notes in the high register are played with a lesser degree of precision with respect to theoretical intonation, the results point towards other factors playing a part, such as these notes' increased difficulty for string instrumentalists[61]. When studying the effects of vibrato on intonation, scholars have faced no noticeable impact[62] and sometimes deem it as an strategy for accuracy perception[63].

While this article aims not at offering a general theory of intonation in performance but rather an explanation for Casals' decisions in a specific interpretation, it nonetheless contributes insights into some of the challenges just reviewed. From a clear performance analysis perspective, it explores an

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[51] Geringer, "Intonational performance…", 36; and Madsen, "The effect of scale direction…", 271.

[52] Duke, "Wind instrumentalists' intonational performance…", 109; and Sogin, "An analysis of string instrumentalists' performed intonational adjustments …", 108. For his part, Greene ("Violin performance with reference…", 243) observed no significant differences.

[53] Edmonson, "Effect of interval direction…", 248-249; and Kantorski, "String instrument intonation…", 206.

[54] Loosen, "Intonation of solo violin performance…", 537; Madsen, "The effect of scale direction…", 271; and Yarbrough and Ballard, "The effect of accidentals…", 20, although to them the results were not conclusive enough.

[55] Rakowski, "Intonation variants of musical intervals…", 70.

[56] Duke, "Wind instrumentalists' intonational performance…", 109; Greene, "Violin performance with reference…", 243; and Loosen, "Tuning of diatonic scales…", 224.

[57] Elliot, Platt and Racine, "Adjustment of successive and simultaneous intervals…", 594.

[58] Rakowski, "Intonation variants of musical intervals…", 61.

[59] *Ibid.*, 63.

[60] Shackford, "Some aspects of perception I…", 190; in the case of whole steps in harmonic contexts, he found an overall deviation of 56 cents.

[61] Garman, "The effects of accompaniment texture…", 90; and Kantorski, "String instrument intonation…", 208.

[62] Geringer and Sogin, "An analysis of musicians' intonational adjustments…", 4; Greene, "Violin performance with reference…", 233; and Sogin, "An analysis of string instrumentalists' performed intonational adjustments…", 109.

[63] John M. Geringer, Rebecca B. MacLeod, Clifford K. Madsen and Jessica Napoles, "Perception of melodic intonation in performances with and without vibrato", *Psychology of Music* 43, no. 5 (2015): 675; and Robin Stowell, *Violin Technique and Performance Practice in the Eighteenth and Early Nineteenth Centuries* (Cambridge: Cambridge University Press, 1985), 254.

entire commercial recording; that is, it explores real performance and not an artificial situation. This recording furthermore provides examples of not only ascending/descending and small/large intervals, but also of contrasting materials (harmonic vs. melodic) within a single performance occasion, as well as of less-explored interval types, such as diminished fifths and fourths or augmented seconds or fourths. It also tries to reconcile the conclusions drawn from the large-scale evaluation of the data, as music psychologists have tended to do, with the passage-particular reality of Casals' practice, as is the custom in performance analysis. Placing an emphasis on the uniqueness of Casals' interpretive practice, it eventually transcends the pure descriptive assessment of interval sizes and cent deviations by inquiring into intonation not as a parameter separate from other performative strategies. Rather, it ultimately explores the potentially structural—and not only expressive—function that intonation, in conjunction with tempo and dynamics, may play in performance. Whether or not this was a conscious decision on Casals' part is beyond the purposes of this investigation.

III. Methods

The analysis is based on data on the deviation (in cents) between the actual intonational measurements and the theoretical values of the notes in the three tuning systems considered in this article, i.e., equal temperament, just intonation, and Pythagorean tuning. The actual intonation values in Casals' studio recording of Bach's E flat major Prelude were extracted, in Herzs (Hz), using the free software Sonic Visualiser. A melodic spectrogram with dBV^2 scale was preferred, as in the lineal scale many notes, especially those in the lower register, are not clearly visible (compare figures 1a and 1b). Values for all the notes in the piece were manually extracted. In the very few instances in which Casals adjusted the frequency of the note throughout its course, normally sharpening it[64], the mean value between the start and the ending frequencies was taken. When the fundamental frequency of the note was not visible or the line in the spectrographic image was too thick, the measurements were made on a higher octave and then divided by 2 or 4[65], as relevant (see figure 2).

[64] This tendency has been noted in other performers. See Fabian, "The recordings of Joachim, Ysaÿe and Sarasate…", 196; Janina Fyk, "Intonational protention in the performance of melodic octaves on the violin", in *Music, Gestalt, and Computing. JIC 1996. Lecture Notes in Computer Science (Lecture Notes in Artificial Intelligence)*, ed. M. Leman (Berlin and Heidelberg: Springer, 1997), 421; and Sogin, "An analysis of string instrumentalists' performed intonational adjustments…", 109.

[65] As table 1 shows, consecutive octaves stand in a frequency ratio of 2:1.

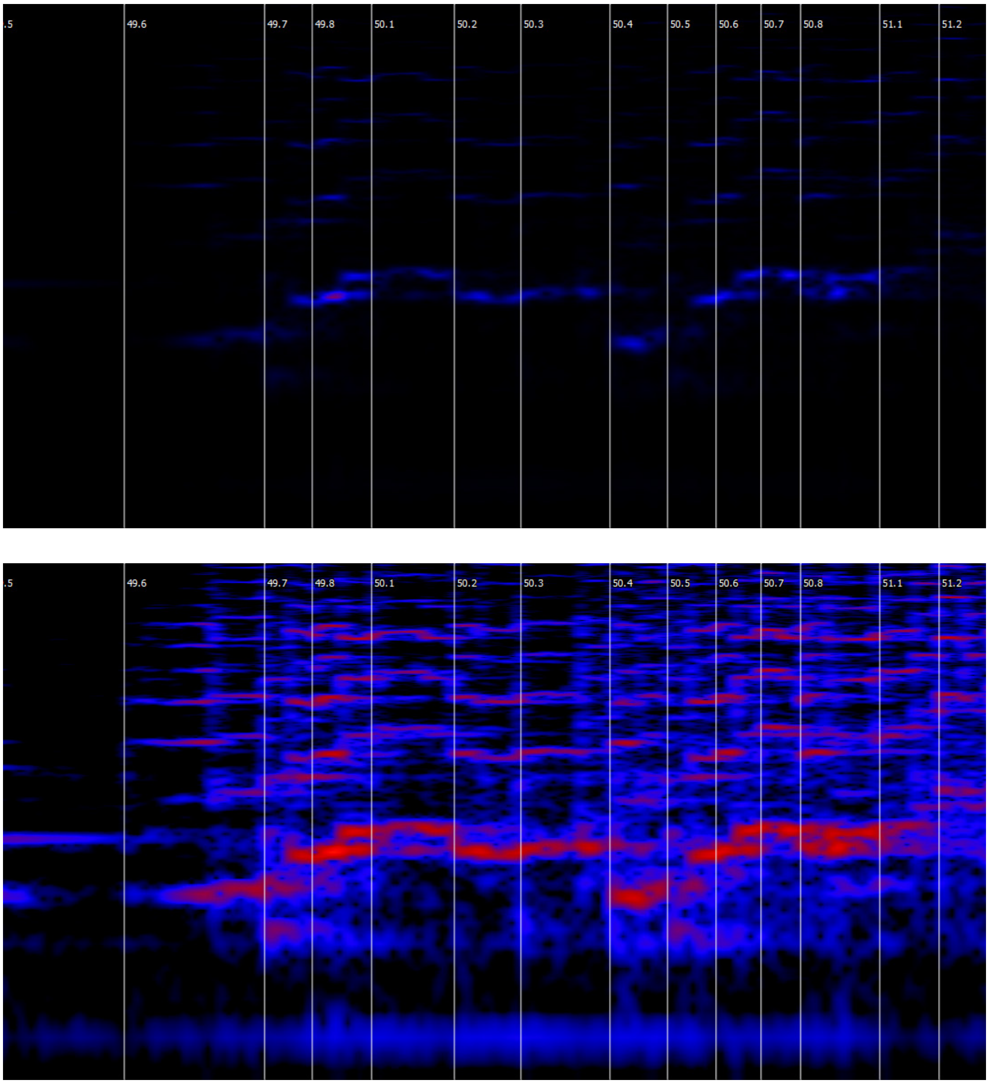

Figure 1. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, bars 49.5-51.2, recording by Pau Casals. Melodic spectrogram: a) linear scale; b) dBV^2 scale

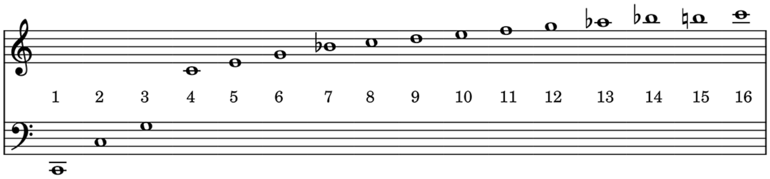

Figure 2. First 16 overtones in the natural harmonic series, over C_2

To calculate the theoretical values of the notes in the three tuning systems, three scenarios were established depending on three alternative reference points: i) the first note of the piece, ii) the note preceding the analysed note, and iii) the fundamental note of each harmony in the piece[66]. In this manner, it was possible to discriminate the passages in which Casals purportedly thought more harmonically or more linearly, and also if his views on intonation were large- or small-scale. The first two scenarios offer no point of discussion, but the definition of the relationship between fundamental chord tones necessarily relies on a system that determines the exact distance between each pair of notes. As the annotated score in Appendix I shows, the harmony in Bach's prelude changes at an average rate of 2 bars, with some passages moving faster or slower harmonically. The circles indicate the notes that have been deemed as the fundamental of the chords[67], and the theoretical values of such fundamental notes were calculated as perfect intervals, i.e., as in the just intonation, with respect to the fundamental note in the previous harmony. Even when in the bass, passing notes were treated as such and not as belonging to the relevant chord.

In each scenario, the theoretical frequency of every note was calculated for the three tuning systems (table 1)[68]. Note that octaves and the unison are the only intervals with the same ratio in the

[66] This approach is similar to the one that Brown adopted in her doctoral dissertation. For the analysis of recordings by renowned violinists of the first 15 notes of Bach's E major violin partita, which, similarly to the cello prelude explored here, opens with a tonic arpeggio, she calculated deviation range and standard deviation with respect to I) each preceding note, ii) the first note of the piece, iii) the fundamental note of the first chord, iv) the fundamental notes of the two chords in the excerpt, and v) the average tonic. In our case here, Brown's scenarios iii and iv would be the same. Although in Brown's study scenario 5 resulted to be the "best description of intonation", it is discarded from the present analysis as it disregards that reference points vary across a piece, as we have discussed above. See Brown, "Dynamics of intonation…", 92. For his part, Ornoy ("Between theory and practice…") adopted a note-to-note approach except in harmonic contexts.

[67] Some passages afford varying interpretations as, for instance, a diminished seventh followed by the dominant seventh of the same harmony: chords could be treated separately or as two manifestations of the same harmony. To avoid confusion, colour lines in the score represent the criteria for harmonic change in this analysis.

[68] For the just intonation and the Pythagorean tuning, frequencies are calculated by ratios, as reflected in table 1, multiplying by the relevant value in the case of ascending intervals and dividing it by it in descending ones. For the equal temperament, a mathematically more complex operation must be performed: $f_t = f_r * \left(\sqrt[12]{2}\right)^n$, where f_t = the theoretical frequency one wants to calculate; f_r = the frequency of the reference note; and n = the number of semitones separating the two notes, with negative values in the case of descending intervals.

Pythagorean tuning, the just intonation, and the equal temperament (2:1 and 1:1 respectively). The just intonation and the Pythagorean tuning also share the value for the perfect fifth (3:2) and the perfect fourth (4:3).

Table 1. Interval ratios in the equal temperament, the just intonation, and the Pythagorean tuning[69]

Interval*	Pythagorean tuning			Just intonation			Equal temperament		
	Numerical origin	Ratio	Cents	Numerical origin	Ratio	Cents	No. of semitones	Ratio	Cents
P1	1:1	1.000	0	1:1	1.000	0	0	1.000	0
m2	$2^8:3^5$	1.053	90.2	16:15	1.067	111.7	1	1.059	100
a1	$3^7.2^3$	1.068	113.7	15:14	1.071	119.4	1	1.059	100
M2	$3^2:2^3$	1.125	203.9	10:9 (lesser) 9:8 (greater)	1.111 1.125	182.4 203.9	2	1.122	200
d3	$2^{16}:3^{10}$	1.110	180.45	19:17	1.118	192.56	2	1.122	200
m3	$2^5:3^3$	1.186	294.1	6:5	1.200	315.6	3	1.189	300
a2	$3^9:2^{14}$	1.201	317.6	15:13	1.154	247.7	3	1.189	300
M3	$3^4:2^6$	1.265	407.8	5:4	1.250	386.3	4	1.260	400
d4	$2^{13}:3^8$	1.249	384.4	13:10	1.300	454.2	4	1.260	400
P4	$2^2:3$	1.333	498.1	4:3	1.333	498.1	5	1.335	500
d5	$2^{10}:3^6$	1.405	588.3	45:32	1.406	590.2	6	1.414	600
a4	$3^6:2^9$	1.424	611.7	64:45	1.422	609.8	6	1.414	600
P5	3:2	1.500	702.0	3:2	1.500	702.0	7	1.498	700
m6	$2^7:3^4$	1.580	792.2	8:5	1.600	813.7	8	1.587	800
a5	$3^8:2^{12}$	1.602	815.6	25:16	1.563	772.6	8	1.587	800
M6	$3^3:2^4$	1.688	905.0	5:3	1.667	884.4	9	1.682	900
d7	$2^{15}:3^9$	1.665	882.4	38:23	1.652	869.2	9	1.682	900
m7	$2^4:3^2$	1.788	996.1	7:4	1.750	968.8	10	1.782	1000
a6	$3^{10}:2^{15}$	1.802	1019.1	23:13	1.769	987.7	10	1.782	1000
M7	$3^5:2^7$	1.900	1109.8	15:8	1.875	1088.3	11	1.888	1100
P8	2:1	2.000	1200.0	2:1	2.000	1200.0	12	2.000	1200

*P = perfect, M = major, m = minor, d = diminished, a = augmented.

[69] This table has been prepared departing from table 1 in Edward M. Burns, "Intervals, scales, and tuning", in *The Psychology of Music*, ed. Diana Deutsch (Cambridge, MA: Academic Press, 2012), 216. Further diminished and augmented intervals have been added, and also some corrections needed to be made.

Once the values for the performed and the theoretical frequencies were obtained for the three tuning systems in the three reference scenarios, Hz values were converted into cents in order to have a scale with comparable meaning for all the regions of the register[70]. In the results, negative cent values indicate that the note as performed is flatter than its theoretical value in the relevant tuning system and scenario, whereas positive values stand for notes that are performed sharper than in the theoretical calculations. To assess whether the intervals are enlarged or narrowed, in the case of descending intervals the results are changed sign. For instance, if the deviation for the second note of a descending fifth is negative, then the note is played flat and, accordingly, the interval is made bigger. For easier understanding, I refer to enlarged intervals as "sharp" and to narrowed ones as "flat", as if all of them would be in an ascending direction.

Results were evaluated by assessing the dispersion of the data, the mean deviations between the performed and the theoretical intonation values, and the standard deviation (sd) thereof. These were calculated for all intervals and interval types (perfect, major, minor, diminished, and augmented). The analysis was performed:

a) In the whole piece
b) For each of the three sections
c) For each of the materials (arpeggios vs. scales)
d) Discriminating by interval direction (ascending vs. descending)
e) For each harmony in the piece.

At the end of the article, reference is made to fluctuations in tempo and dynamics in Casals' performance of Bach's prelude. To retrieve such data, the recording was manually tapped in Sonic Visualiser at the quaver level, i.e., 8 measurements per bar. Data on the dB per beat were subsequently obtained through the online application Dyn-a-matic[71]. To align cent deviation values, measured for every note, with tempo and dynamic data, retrieved for every quaver only, mean values for the three parameters were aggregated for each of the harmonies in the piece, thus allowing for diachronic representation over an x axis. Following a customary procedure in statistics, this also prevented the data from seeming too variable and idiosyncratic on the small scale. Given the apparently inconsequential impact of vibrato on intonation, as discussed in the previous section, and since this resource is pervasive in Casals' recording of Bach's E flat major prelude, no discrimination between notes with and without vibrato is made in the analysis.

[70] As frequencies follow a logarithmic principle, with octave frequencies being calculated by multiplication instead of by sum of a constant value, a linear scale is necessary. In this manner, all octaves can be divided into 1200 cents and, therefore, all semitones occupy 100 cents. This is the mathematical principle underlying the equal temperament. For the conversion, the formula $f_p - f_r(cents) = 1200 * \log_2\left(\frac{f_p}{f_r}\right)$ was applied, where f_p = the performed, real frequency of the note.

[71] "Dyn-a-matic", CHARM–Mazurka project, accessed March 18, 2020, http://www.mazurka.org.uk/software/online/dynamatic/.

IV. Analysis

IV. 1. Tuning systems

The first scenario, i.e., with respect of the first note, seems not to offer a good explanation for Casals' approach to intonation (table 2)[72]. As Geringer noted in the case of other performers[73], if we were to measure all the frequencies with respect to the opening one, "mistuning" would seem to accumulate exponentially—although in reality, in Casals' case this increase does not take place in a constant manner: we can still intervals far removed from the opening of the piece that are nonetheless "in tune" with respect to the first note in the recording. Yet this is not how perception or musical practice work: even if we conceive of music in a balance between the small and the large scales[74], the diachronicity inherent to music, and to music as performed even more[75], makes us perceive the sounds in the small scale, i.e., in respect of the preceding note—as Casals himself commanded[76]—or, at the most, in groups of notes belonging to the same harmony. Sonic memory is short-lived[77] and, thus, a few bars in the performance we forget the absolute frequency of the first E flat in the prelude. Therefore, data have confirmed the musical intuition that scenario i should be henceforth discarded from the analysis. In average, the intervals in the two remaining scenarios—and, indeed, in scenario i too—conform equally closely to the Pythagorean intonation and to the equal temperament, even if Casals explicitly advocated against the latter[78].

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[72] Furthermore, it is not as useful as the others for analytical purposes. As the first note of the piece is a Eb2, just 3 notes, D2, Db2, and C2—and their enharmonic sounds—would stand in a descending-interval relationship to it. Hence scenario i's advantage for some of the analyses performed in this study.

[73] There is a "note-by-note increase in absolute cent deviation across unaccompanied performance and perception". Geringer, "Intonational performance and perception…", 38.

[74] "While playing, the performer engages in a continual dialogue between the comprehensive architecture and the 'here-and-now". John Rink, "Translating musical meaning: The nineteenth-century performer as narrator", in *Rethinking Music*, ed. Nicholas Cook and Mark Everist (Oxford: Oxford University Press, 1999), 218.

[75] Cook, *Beyond the Score*…, 70: music does not simple take "place *in* time"; rather, it is "made *of* time".

[76] Especially when he spoke of the notes as forming a chain; see note 4 above.

[77] "…the memory is extremely weak in storing fragments of heard music in a literal way" and, in fact, er draw "out the expressiveness of the music's note-to-note progression". Respectively, Bethany Laura Lowe, "Performance, analysis and interpretation in Sibelius's fifth symphony" (doctoral thesis, University of Southampton, 2000), 60, https://ethos.bl.uk/OrderDetails.do?uin=uk.bl.ethos.365710; and Cook, *Beyond the Score*…, 193.

[78] See note 10 above.

Table 2. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, recording by Pau Casals. Average deviation (in cents) from the three theoretical tuning systems in the three scenarios, for both directional and absolute intervals

	Scenario i (1st note)		Scenario ii (preceding note)		Scenario iii (fundamental of the chord)	
	Directional	Absolute	Directional	Absolute	Directional	Absolute
Equal temperament	26.603	28.409	-0.309	12.775	0.430	10.407
Just intonation	30.853	33.565	-2.249	18.102	3.005	13.548
Pythagorean tuning	22.523	24.027	1.093	13.102	1.384	11.103

It appears, then, that, as Johnson proposed for the c minor sarabande[79], in this performance too Casals' normative intonational custom was closer to the equal temperament than he himself claimed. However, although the mean deviations seem to indicate such a tendency in two scenarios (ii and iii), this is due not to an overall practice but rather to the cellist's approach to specific intervals in particular contexts, as will be discussed below. This is reflected in the dispersion plots—or boxplots[80]—for the absolute intervals in, for instance, the scenario ii (figure 3): while the level of dispersion and the magnitude of the unusually "out-of-tune" notes is comparable for the equal temperament and the Pythagorean tuning, the deviation values are more differential and, on occasions, markedly large, for the just intonation (in the centre of each box group in the plot). Therefore, except for relevant cases, the subsequent discussion will mostly refer to Casals' deviation from the theoretical equal temperament and the Pythagorean tuning.

[79] Johnson, "'Expressive intonation' in string performance…", 83; see theoretical discussion above.

[80] The boxplots in this article represent the dispersion of the absolute intervals. Within each of them, the central rectangles contain the 50% of the data, with the horizontal lines inside them representing the median, i.e., the middle value in the ordered set of deviations. The higher the rectangle—or box—, the more dispersed the intervals are; in other words, the higher the rectangle, the more varied the theoretical dispersions are for the relevant intonation system. The remaining values extend across the lines—or whiskers—up to the most extreme ones. One-instance values are represented unfilled circles, meaning that they are outliers or atypical values.

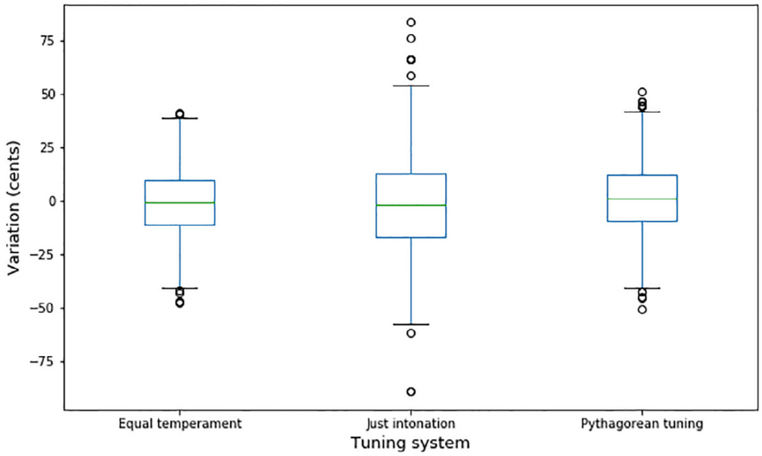


Figure 3. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, recording by Pau Casals. Absolute dispersion of the intervals with respect to the equal temperament, the just intonation, and the Pythagorean tuning; measurements with respect to the preceding note (scenario ii)

This is also reflected very clearly in Casals' intonational approach to the various materials in the piece. Given Casals' supposed preference for the just intonation in double stops, one would expect this intonation system to provide a relatively good model for the cellist's playing in chords and arpeggios too. Yet it resembles his practice worse than any of the two other systems for the two types of materials. For their part, in average the equal temperament and the Pythagorean tuning explain Casals' arpeggios equally well[81]. In fact, Casals' intonation seems to be more varied and potentially "expressive" in the scales, as the difference between the Pythagorean tuning and the equal temperament in such passages is smaller than in the arpeggios.

The sign of the values for the deviation of the directional intervals in table 2 furthermore indicate that, when observed vertically, Casals tends to play slightly enlarged intervals. In this way, the noted tendency is in line with previous "sharpness" findings in the literature[82]. However, when measured from one note to the next, not only do variations became smaller: in respect of the equal

[81] The mean deviation for absolute intervals even points to the equal temperament as a slightly better explicator of Casals' tuning practice in the prelude in scenarios ii and iii for both arpeggios and scales, although the differences are minimal again:

Arpeggios: ii) ET = 9.737, JI = 12.352, PI = 10.573; iii) ET = 12.103, JI = 17.710, PI = 12.354.

Scales: ii) ET = 11.848, JI = 14.173, PI = 12.241; iii) ET = 14.220, JI = 18.944, PI = 14.407.

(ET = equal temperament, JI = just intonation, PI = Pythagorean tuning).

[82] See note 42 above.

temperament and the just intonation, they indicate a certain intonational flatness. This may be due not only to a larger deviation for specific intervals, but also the more marked presence of some of them. In any case, what appears to be clear is that the context and the quantification criteria in musicology—as in any other discipline—must be evaluated and explicated, as varying criteria may result in hardly interpretable results. In our case, for the above reasons, the discussion is restricted to scenarios ii and iii, and we take the equal temperament and the Pythagorean tuning as points of departure to assess Casals' intonational practice in Bach's prelude.

IV. 2. Intervals and interval types

The "compromise between the expressive intonation and the 'tempered' one" of which Casals spoke is in his recording of Bach's E flat major Prelude not a smoothed approach to the intervals: it is not a "compromise" *between* but a compromise *for* the two. While he plays some intervals very close to the equal temperament, others—especially major and augmented—sound notably sharper, i.e., more "Phythagorean".

In theory, minor thirds are complementary to major sixths, and major sixths to minor sixths. One would expect that, to make a perfect octave, if the third of the chord is played flat, then the theoretically complementary sixth would be sharp. Also one would expect major thirds to be played flat so to exploit the natural resonance of the strings in consonance with the just intonation (386 vs. the 400 cents of the equal temperament; see table 1), especially in a major-mode key in which only the third of the chord can be played in an open string in the cello. The first of these suppositions corresponds to Casals' actual performance practice of major thirds. Yet the cellist turns the tables of theory in the second.

This is due to his tendency to play tempered, and on occasions Pythagorean, major thirds. Whether or not Casals was aware that such ample major thirds were in consonance with the composer's own tuning system[83] will remain unanswered. What is clear is that he did not opt for the "pure" major thirds of previous generations of players, in neither arpeggiated nor scalar passages. Interestingly, vertically measured major thirds tend to be similar among themselves more than melodic ones, which indicate that Casals may have set a more stable framework for the harmonic blocks in the piece (scenario iii) while playing more with ampler, Pythagorean, major thirds from one note to the next within the arpeggios (scenario ii), only to obtain perfect fourths, fifths, and octaves to frame such internal intervals (iii). Note, in figure 4a (ii), how the mean deviation for the internal, melodic major thirds within the arpeggios are closer to the Pythagorean theory and how they are much sharper than the just counterpart. In the scales, the few major thirds are not as wide. When measured harmonically (figure 4b, iii), the thirds of the chords are virtually tempered, in both arpeggios and scales. They are

[83] See note 46 above.

moreover compensated by slightly flat consecutive minor sixths (ii) to make a perfect octave (iii), and by small minor thirds too to make a perfect harmonic fifth. For their part, these small minor thirds (measured both melodically and harmonically, ii and iii), are compensated by ample major sixths (ii) to make a perfect octave (iii) and by sharp, non-pure, major thirds to make a perfect fifth.

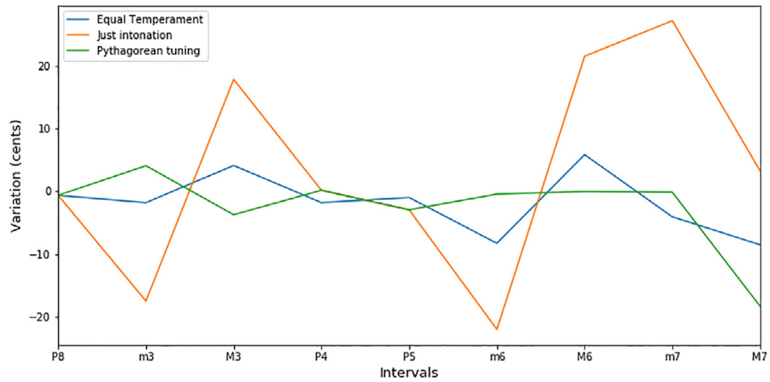


Figure 4a. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, recording by Pau Casals. Mean deviation of specific directional intervals with respect to the equal temperament, the just intonation, and the Pythagorean tuning; harmonic measurements (scenario iii)

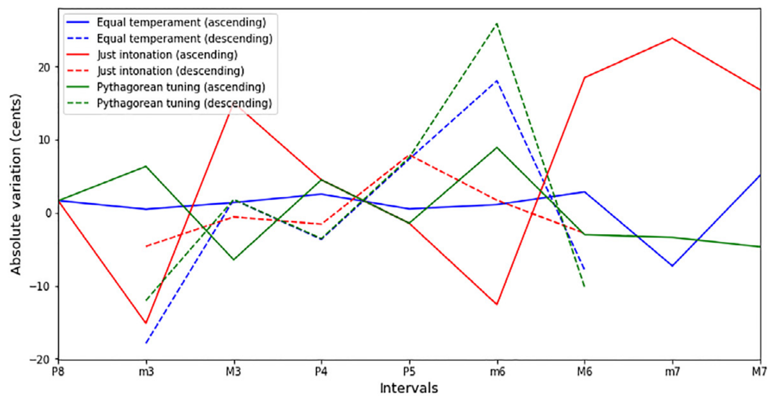


Figure 4b. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, recording by Pau Casals. Mean deviation of specific directional intervals with respect to the equal temperament, the just intonation, and the Pythagorean tuning; note-to-note measurements (scenario ii)

In the opening bars (bars 1-2), this is already apparent (figure 5). Casals plays the third of the major chord (G_3) "sharp", even sharper than the Pythagorean theoretical type, with respect of the fundamental of the chord (Eb_2). Then, this third is followed by a small minor third in such a way that the perfect fifth between this resulting note and the bass of the chord (Eb_2-Bb_3) is closer to the just type than any of the thirds. Bs. 62-63 (figure 6) also epitomise the phenomenon of the compensating non-just stacked thirds in a minor-chord context. In it, the G_2-Bb_2 third is flat, whereas the major third Bb_2-D_3 is sharper than the just type to make a G_2-D_3 fifth that is almost pure. In both passages, perfect intervals (P4, P5, and P8) tend slightly to sharpness, even when some of their notes are played on open strings (G_2-D_3 in bs. 62-63).

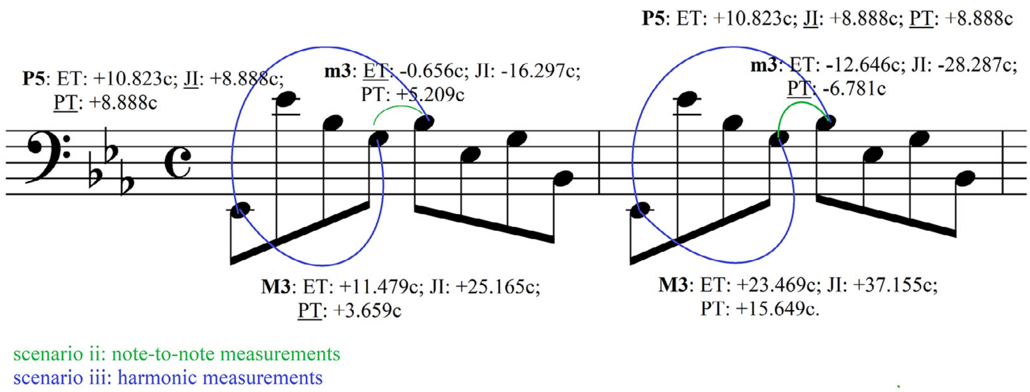


Figure 5. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, bs. 1-2, recording by Pau Casals. Cent deviation for the intervals with respect to the equal temperament, the just intonation, and the Pythagorean tuning; melodic and harmonic measurements (scenarios ii and iii)

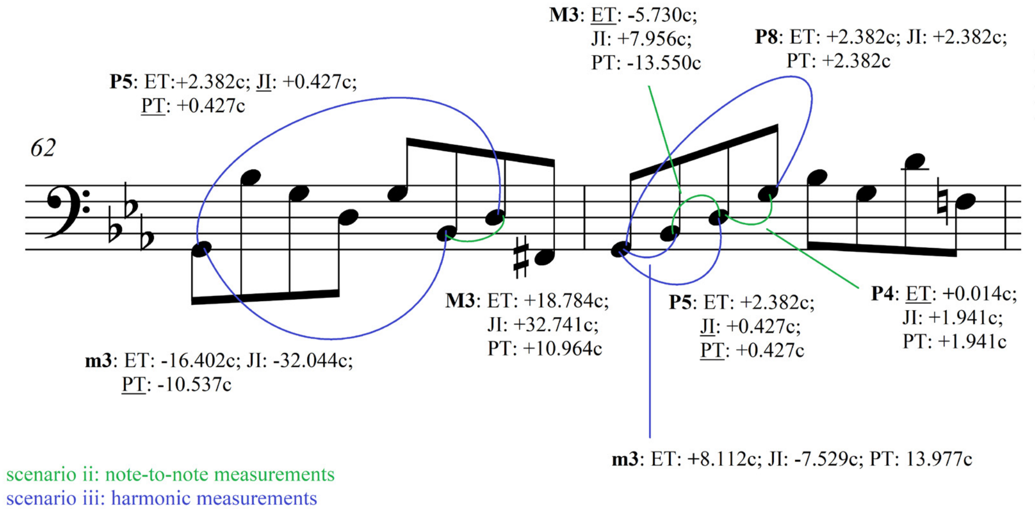


Figure 6. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, bs. 62-63, recording by Pau Casals. Cent deviation for the intervals with respect to the equal temperament, the just intonation, and the Pythagorean tuning; melodic and harmonic measurements (scenarios ii and iii)

The tendency towards tempered intervals in the first place, and to Pythagorean sizes in the second, is especially evident in the case of "less-normative" or more expressive, intervallic types. These emerge in the prelude more in scenario iii, i.e., as part of chords that can materialise in the form of both arpeggios and scales over a diminished or an augmented harmony, such as in bars 49ff. In the arpeggios, diminished intervals are closer to equi-tempered values, whereas in the scales they tend more to Pythagorean sizes (figure 7). As a norm, Casals plays the augmented ones sharp, in both arpeggiated and scalar passages.

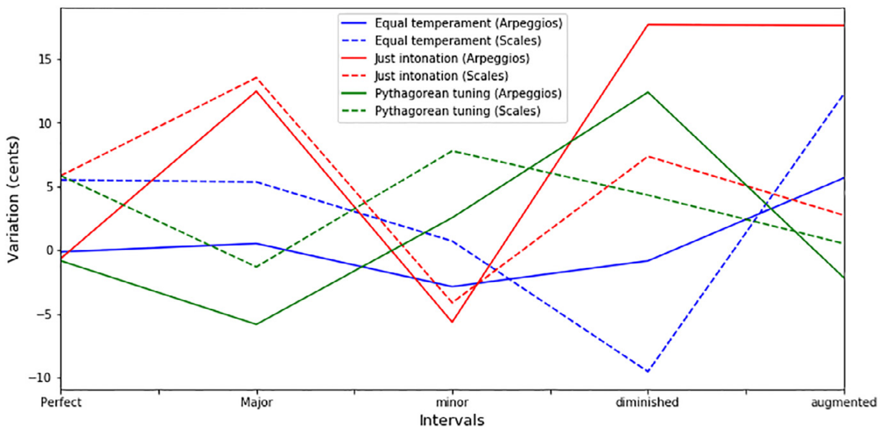


Figure 7. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, recording by Pau Casals. Mean deviation of the five interval types (directional) with respect to the equal temperament, the just intonation, and the Pythagorean tuning; note-to-note measurements (scenario ii)

Whereas in harmonic contexts Casals seeks for a compromise in diminished intervals, in the melodic sections of the piece he opts for a more expressive intonation in which they are "flat", these resulting from expressive uses of minor and major seconds too. In bs. 33-34 (figure 8), for instance, except the bigger intervals at the start of bar 33, all diminished types are narrowed with respect to the equi-tempered values. In fact, Casals' diminished sevenths in this excerpt are notably close to the also flat Pythagorean sizes, whereas, as a whole, diminished fifths are comparatively wider. That is, the second and the third minor third of the diminished chord, that between degrees $\hat{3}$ and $\hat{5}$ and , is much smaller and thus provides the passage with its diminished flavour—as a matter of fact, in this passage it becomes smaller as the whole chord becomes narrower too (figure 8b). The C_3 that functions as the leading note towards the seventh of the chord is sharpened in a way that the melodic semitone is 32.290 cents smaller than the equi-tempered one. For an easier visualisation, in figures 8a and 8b, as well as in subsequent ones, cent deviation values are provided with respect to the equal temperament only.

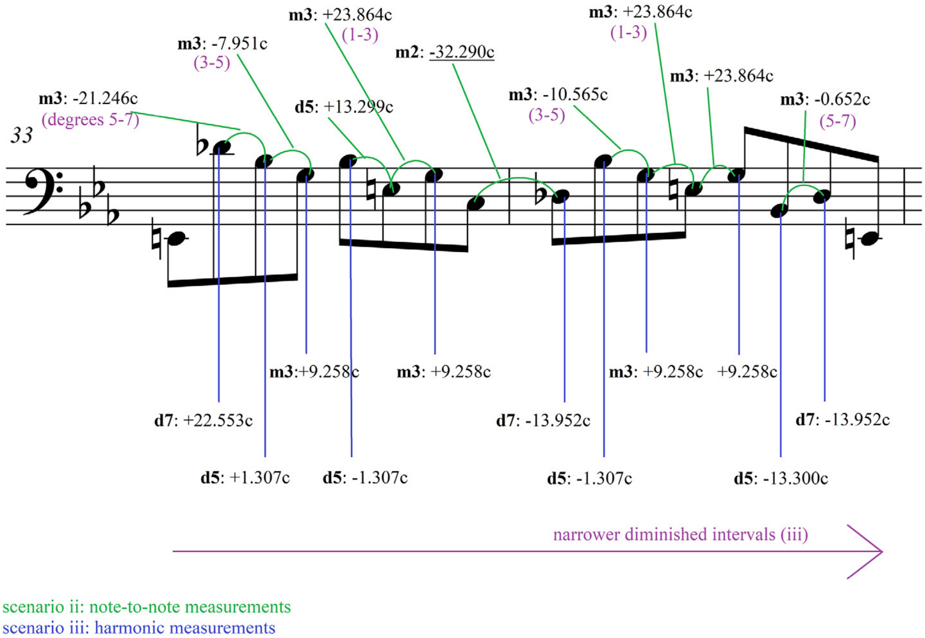


Figure 8a. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, bs. 33-34, recording by Pau Casals. cent deviation for the intervals with respect to the equal temperament; melodic and harmonic measurements (scenarios ii and iii), diminished-chord arpeggio

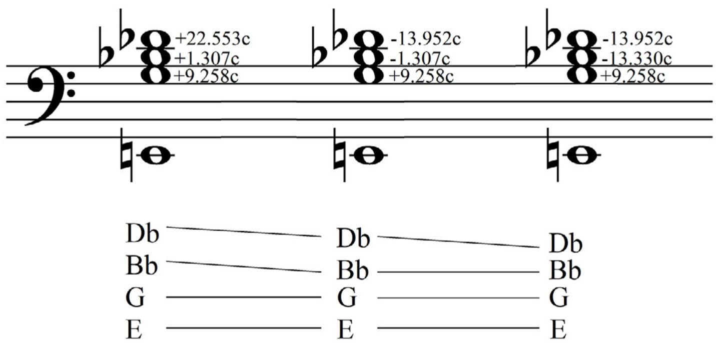


Figure 8b. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, bs. 33-34, recording by Pau Casals. Progressive narrowing of the diminished intervals of the chord (scenario iii). Reduction

The same phenomenon occurs in melodic passages over diminished harmonies too. In bs. 49-51 (figure 9), it is the third between degrees $\hat{3}$ and $\hat{5}$ and, in this case between E and G, that is most narrowed in order to produce diminished fifths close to the tempered values and diminished sevenths that, as a consequence of the succession of the intervening seconds, become flatter as the notes elapse. It also seems that, again, Casals sought for expressive diminished sevenths, as the impression of the chord and its natural overtones was not as enduring as in the arpeggio and therefore he could afford larger deviations. In fact, as the melodic line goes upwards, the intervals get narrower not through the flatness of the higher notes but through the sharpness of the lower ones. The intermediate changes are a consequence of the narrowness of diatonic semitones (as between $F\#_3$ and G_3 in bar 50) or of the changes in the direction of the melody itself, as at the end of bar 50. In spite of these inconsequential changes, we again have a narrow sonority, in this case away from the low $C\#_2$ that functions as the bass of the chord (figure 9b) and not getting closer to the bass, as in figure 8. As the melodic design goes downwards from bar 51iii, the intervals are narrowed again. In other words, Casals' intonational strategy in this passage follows the direction of the melodic design: as in other diminished contexts, the sonority progressively becomes narrower, away from the bass if the melody moves upwards (see upward vertical arrows in figure 9b) and closer to the bass if the melody returns to it (see downward arrows). In the first part of the passage, the notes revolve, "sharp", around the Bb_3, moving away from it when the scale goes down; in the second and especially near the end, the most stable note is the G_3, which leads into the D_2 through an almost perfect fourth. The diminished third (plus 2 octaves) between the initial $C\#_2$ in bar 49 and the high Eb_4 in bar 51 is very flat too, being produced by a very much narrowed melodic minor third.

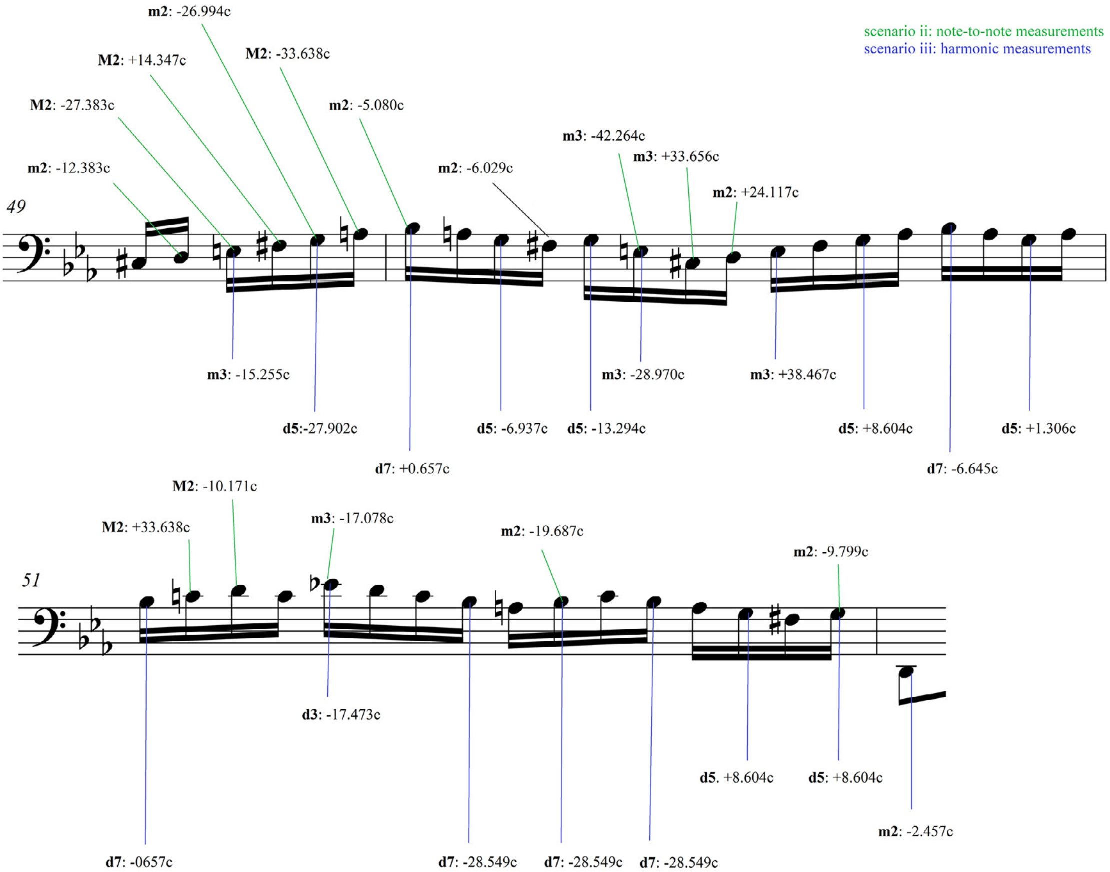


Figure 9a. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, bs. 49.6-52.1, recording by Pau Casals. Cent deviation for the intervals with respect to the equal temperament; melodic and harmonic measurements (scenarios ii and iii), scale over diminished harmony

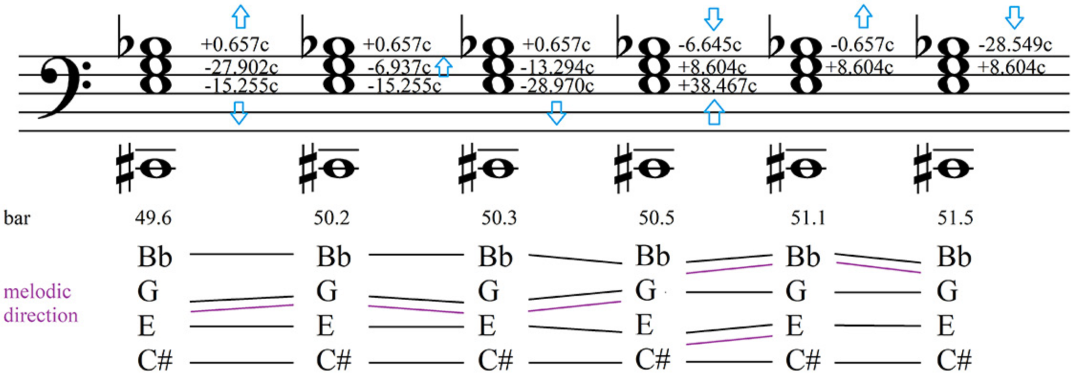


Figure 9b. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, bs. 49.6-52.1, recording by Pau Casals. Progressive narrowing and enlarging of the diminished intervals of the chord in consonance with melodic direction (scenario iii). Reduction

Casals' augmented intervals in the prelude are, on the contrary, sharp, as in bar 56 (figure 10). This is perceptually allowed by the initial major seventh (plus an octave), which is enlarged in such a way that the minor sixth D2-Bb3 is just slightly flatter than the by-nature-large just interval (-3.262 cents). Also, melodically that major seventh would resolve upwards, eventually producing a very small, Pythagorean diatonic semitone. Moreover, in the subsequent bars, minor thirds progressively become larger as they correspond to the lower notes of the chord, in a way similar to the manner in which the upper thirds of the diminished harmony in bs. 49-51 are flatter than the lower ones (see the light blue arrows in figure 10a). In fact, the descending diminished chord C#$_3$-E$_3$-G$_3$-Bb$_3$ in bs. 56-58 has the same conformation as that in bs. 49-51: a comparative larger minor third at the bottom over which a smaller one gives the diminished flavour. For its part, the upper one, between degrees $\hat{5}$ and $\hat{7}$ and , is larger than the preceding one but narrow in comparison to the one over the fundamental note. To these, a small major third is added at the bottom to produce an almost perfect fifth between E$_3$ and A$_2$, and similarly the largest minor third is also added as the intonation as a whole goes flat to balance the sharpness of bar 56 (figure 10b). This produces a sense of a progressively expanded chord that resolves into the following harmony (bar 60) through a large—or "greater", see table 1—descending major second.

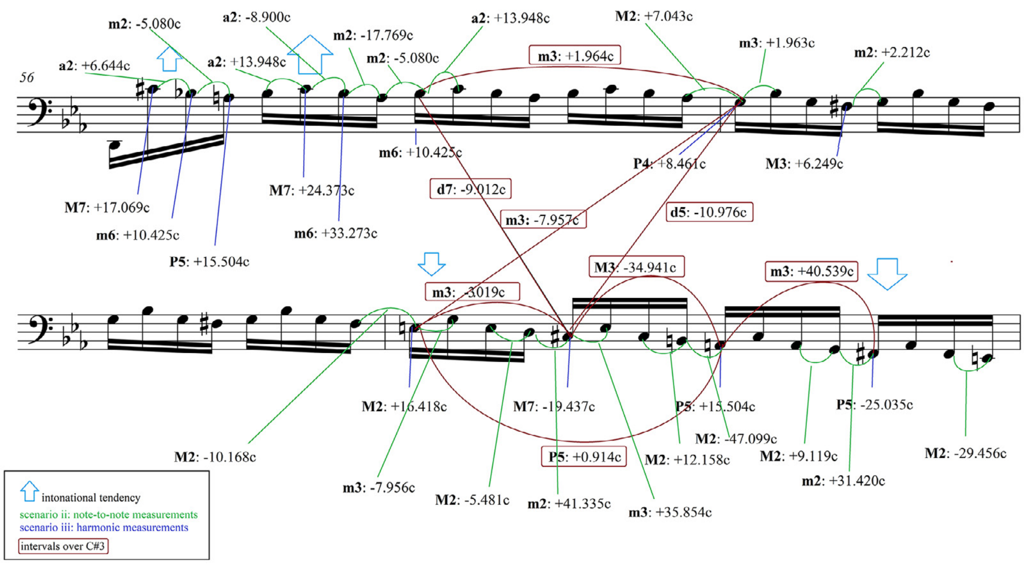


Figure 10a. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, bs. 56-58, recording by Pau Casals. Cents deviation for the intervals with respect to the equal temperament; progressive enlargement of the minor thirds as the melody goes down (scenario ii)

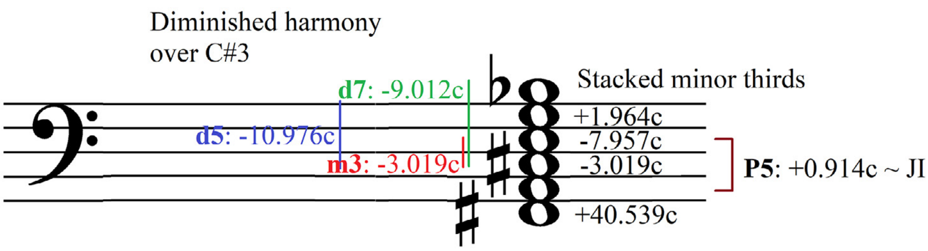


Figure 10b. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, bs. 56-58, recording by Pau Casals. Conformation of the diminished chord over C#$_3$ and addition of lower thirds. Reduction

Bs. 56-58 form one of the passages in which the larger deviations from theoretical values occur. As annotated in figure 10, major seconds in Casals' recording deviate as much as -47.099 cents from the theoretical equi-tempered value (bar 58ii-iii), and, similarly, minor thirds as much as +35.854 cents (bar 58i). There is also a minor second that is 41.335 cents sharper than the 100-cent equi-tempered type (D_3-$C\#_3$, bar 58i-ii) and a major third that deviates -34.941 cents from the theoretical value (bar 58ii). Casals therefore plays both major and minor intervals sharp and flat alike. Interestingly, such larger deviations occur at the end of the passage, i.e., in its way towards the resolution and necessary adjustment after its sharp tendency at the outset.

Although the statistical evaluation of the data indicates that, with respect to both the fundamental note of the chord and the preceding one, intervals in diminished contexts tend to be sharp (figure 11), the above passage features minor thirds and diminished fifths and sevenths that are narrowed too. This apparent incongruence between small- and large-scale analysis can be bridged by looking at each interval type within each harmonic context. In the case of diminished harmonies, the average cent deviation for minor intervals—normally minor thirds over the fundamental note—is +4.520 cents in respect of the theoretical value for the equal temperament, and for diminished types—both fifths and sevenths—the average deviation is -2.198 cents. In other words, the first minor third over the fundamental note of the chord is large and the vertically successive ones are small, in consonance with the "progressively narrowed" phenomenon observed in the two passages above. It is the presence of sharpened passing notes that produces the average deviation towards sharpness, not the notes of the chord themselves.

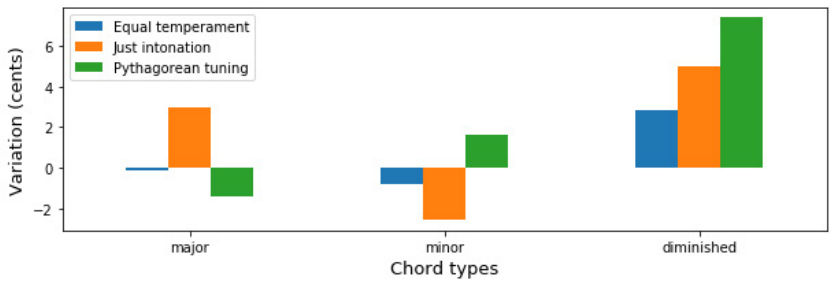


Figure 11. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, recording by Pau Casals. Mean deviation of intervals with respect to the equal temperament, the just intonation, and the Pythagorean tuning; harmonic measurements (scenario iii) for each chord type separately

In major contexts, intervals are, in average, slightly flat in relation to the equi-tempered value, in a sort of compromise between the just and the Pythagorean systems. The same can be said of minor contexts, in which flatness is more pronounced and the sense of compromise is equally descriptive. As expected, in both harmonic contexts thirds are closer to Pythagorean values, meaning that minor thirds are narrow and major ones are wide.

IV. 3. Interval direction and interval size

Overall, there is no notable difference in the way in which Casals tunes the ascending and the descending intervals. If the dispersion of descending intervals in scenario iii is lesser, this is due to the fact that these are proportionally fewer than the ascending ones. In terms of mean variation, measured melodically intervals in both directions show a similar tendency towards flatness (-0.294 and -0.319 cents respectively; see table 2 for congruent results).

As it has just been exemplified, however, when observed in the context of specific passages some tendencies become apparent. In figure 9a, the intonation fluctuates between sharpness and flatness as the melody goes up and down. In bar 50ii especially, the E_3 is flat because it goes down, and the $C\#_3$ goes up because the melody starts to go up and semitones function as local leading notes (in this case, the D_3 is played as an open string, thus, overruling the fact that the C# is the bass of the chord). In fact, as the intervals become sharper as the melody goes upwards, the large descending minor third E_3-$C\#_3$ has the function of also producing an almost perfect octave $C\#_3$-$C\#_2$ (the latter in bar 59). In figure 10, the same occurs: the big leap upwards at the start pushes the intonation high, yet the overall descending motion of the melody afterwards returns everything in place, so to speak.

That is to say, it is not the direction of a particular interval that determines the direction of its deviation. As we have seen, both ascending and descending intervals have, in average, the same tendencies and, furthermore, Casals plays a specific interval, for instance an ascending minor third, both enlarged and narrowed[84]. Rather, it is the direction of the medium-scale melody that determines the direction of such deviation. That is why no clear rationale can be found for the arpeggiated sections in the prelude: the arpeggios move up and down for short periods of time and, thus, no sense of melodic direction is established. The scales are, on the contrary, good examples to understand Casals' intonational strategy and his reaction towards direction.

This phenomenon is in relation to string players' tendency to adjust notes sharp as these are higher in the register, as noted in the literature apropos violinists and cellists[85] as well as in the general performance practice—we normally play sharper in the high positions of the fingerboard. The rule seems to be: "the highest the note, the sharper it *must* be played", although in real practice, or at least in Casals', this seems to apply to passages in which the register remains more or less constant, such as scales and not in the case of alternating arpeggios.

This E flat prelude by Bach is a good case study on interval size as a factor for tuning preferences. Virtually all of the arpeggios start with a large ascending leap, only to progressively and indirectly descend through the arpeggio across the bar. By comparison, Casals tends to play small

[84] For instance, 0.656 cents in bar 1 (G_3-Bb_3) and +9.261 cents in bar 3 (the same G_3-Bb_3), with respect to the equal temperament in both cases.

[85] Garman, "The effects of accompaniment texture…", 89.

melodic intervals (up to the major seventh, in scenario ii) slightly sharp (+0.182 cents with respect to the equal temperament) and large ones (from the perfect octave), flat (-5.340 cents). The absolute values also indicate a smaller deviation in the small intervals (12.636 cents) than in the larger types (14.286 cents); in fact, large intervals form the group that accommodates better to the theoretical values of the just intonation (-0.484 cents). Interestingly, major thirds are the only intervals that are more "in tune" when they expand over more than one octave (5.999 absolute cent deviation) than when they are reduced to the 4- and 9-semitone intervals (13.302 cents). The remaining intervals show a lesser absolute deviation when within the octave, thus accounting for the general tendency.

As said above, the few big ascending leaps in the melodic passages have the function of pushing the intonation upwards in a way that the sharpness is compensated before the resolution into the following harmony (figure 10a) and, therefore, these intervals' enlargement should not come as a surprise. In the arpeggios, on the contrary, the first leap of each bar tends to be narrowed, hence the general flat tendency for large intervals in the recording. Yet, as shown in figure 8a, sometimes Casals plays those leaps very sharp. Table 3 contains the measurements for the large leaps in the arpeggiated passages in the piece. The grey tonalities highlight sharp deviations, the darker ones signaling those larger than +10 cents.

Table 3. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, recording by Pau Casals. Deviation (in cents) of large intervals from the three theoretical tuning systems; note-to-note measurements (scenario ii)

			Cents deviation				
Beat	Hz	Interval	Equal temperament	Just intonation	Pythagorean tuning	Chord	Leads into degree
1.2	307.79	2 * P8	0.000	0.000	0.000	I	$\hat{1}$
2.2	310.40	2 * P8	9.376	9.376	9.376	I	$\hat{1}$
3.2	276.578	m7 + P8	-14.346	16.828	-10.436	I7	$\hat{7}$
4.2	275.414	m7 + P8	-38.856	-7.682	-34.946	I7	$\hat{7}$
5.2	264.439	M6 + P8	10.035	25.676	4.170	IV	$\hat{3}$
6.2	265.556	M6 + P8	5.343	20.984	-0.522	IV	$\hat{3}$
7.2	207.621	P4 + P8	-13.438	-11.483	-11.483	V7	$\hat{7}$
8.2	208.813	P4 + P8	-18.121	-16.166	-16.166	V7	$\hat{7}$
9.2	196.545	M3 + P8	1.565	15.251	-6.255	I	$\hat{3}$
10.2	197.375	P4 + P8	8.464	10.419	10.419	I	$\hat{3}$
11.2	313.031	m3 + 2*P8	-3.268	-18.909	2.597	vi	$\hat{3}$
12.2	313.031	P4 + P8	20.995	22.950	22.950	vi7	$\hat{3}$

13.2	310.401	d5 + P8	-8.610	1.474	3.120	V7/V	$\hat{7}$
14.2	311.713	m7 + P8	-16.958	14.217	-13.048	V7V	$\hat{7}$
15.2	296.331	M3 + P8	16.166	29.853	8.346	V	$\hat{3}$
16.2	274.255	P4 + P8	-28.044	-26.089	-26.089	v	$\hat{3}$
17.2	274.995	d5 + P8	-31.064	-20.980	-19.334	V7/IV	$\hat{7}$
18.2	275.414	m7 + P8	-16.957	14.217	-13.047	V7/IV	$\hat{7}$
19.2	263.326	M3 + P8	-3.124	10.562	-10.944	IV	$\hat{3}$
20.2	264.04	P4 + P8	8.471	10.426	10.426	IV	$\hat{3}$
21.2	207.934	m3 + P8	-32.471	-48.112	-26.606	vii7/vi	$\hat{7}$
22.2	208.813	P4 + P8	-13.434	-11.479	-11.479	iv/vi	$\hat{3}$
23.2	208.499	d5 + P8	-11.217	-1.132	0.513	V9/vi	$\hat{9}$
24.2	206.188	m6 + P8	-26.082	-39.768	-18.262	iv/vi	$\hat{3}$
25.2	293.399	m3 + P8	-13.183	-28.824	-7.318	vii7/vi	$\hat{3}$
26.2	296.331	P5 + P8	-6.385	-8.340	-8.340	V7/vi	$\hat{5}$
27.2	311.713	m3 + 2*P8	-17.870	-33.511	-12.005	vi	$\hat{1}$
28.2	313.031	m3 + 2*P8	-17.884	-33.526	-12.019	vi	$\hat{1}$
29.2	330.672	a4 + P8	-11.215	-20.721	-22.945	V7/ii	$\hat{3}$
30.2	332.069	a4 + P8	-18.523	-28.029	-30.253	ii	$\hat{1}$
31.2	350.786	M6 + P8	-1.944	13.697	-7.809	ii	$\hat{5}$
33.2	281.707	d7 + P8	22.553	53.315	40.148	V9/ii	$\hat{9}$
35.2	234.986	P4 + P8	-8.206	-6.251	-6.251	ii	$\hat{4}$
36.2	209.696	P4 + P8	-8.207	-6.252	-6.252	ii7	$\hat{3}$
37.2	209.696	d5 + P8	9.512	19.596	21.242	V	$\hat{7}$
39.2	196.841	M3 + P8	-5.744	7.943	-13.564	I	$\hat{3}$
45.2	232.031	m3 + P8	-14.215	-29.856	-8.350	iii	$\hat{3}$
46.23	232.031	P4 + P8	-47.334	-45.379	-45.379	iii	$\hat{7}$
47.2	231.055	P5 + P8	-28.297	-30.252	-30.252	iii	$\hat{5}$
48.2	232.031	m6 + P8	-33.380	-47.066	-25.560	iii	$\hat{5}$
52.2	266.678	m7 + P8	22.153	53.327	26.063	V7/iii	$\hat{7}$
53.2	264.439	m7 + P8	14.859	46.033	18.769	V7/iii	$\hat{7}$
54.2	233.997	m6 + P8	-4.177	-17.864	3.643	iii	$\hat{3}$
55.2	233.012	m6 + P8	-21.387	-35.074	-13.567	iii	$\hat{3}$
62.2	234.986	m3 + P8	5.506	-10.135	11.371	iii	$\hat{3}$
64.2	275.829	d5 + P8	-23.210	-13.126	-11.480	vii7/ii	$\hat{7}$

66.2	263.326	M3 + P8	-8.346	5.340	-16.166	ii	$\hat{5}$
68.2	307.794	d5 + P8	-23.204	-13.120	-11.474	vii7/V	$\hat{5}$
82.2	311.71	2 * P8	-12.525	12.525	12.525	I	$\hat{1}$
83.2	311.71	2 * P8	-2.614	-2.614	-2.614	I	$\hat{1}$
84.2	278.166	m7 + P8	2.873	34.047	6.783	I7	$\hat{7}$
85.2	275.829	m7 + P8	-16.958	14.216	-13.048	I/	$\hat{7}$
86.2	264.439	M6 + P8	10.035	25.676	4.170	IV	$\hat{3}$
87.2	266.678	M6 + P8	17.331	32.973	11.466	IV	$\hat{3}$

Such very sharp intervals are of all types: minor, major, and diminished, so, at first sight, no theory seems to hold for Casals' practice. Yet, as in the bar 33 of figure 8a, it happens that all such leaps resolve into either the third or the seventh degree of the chord in question, no matter what the actual interval leading into it is. Importantly too, the underlying harmony is in those junctures, with a few exceptions, either the tonic chord, its dominant/leading-note chord, or a tonicisised one, such as g minor in the central part of the piece. That is, Casals plays sharp the third and seventh degrees of the harmonically most important chords of the prelude, in this way creating the sense of a sort of organised, structured intonational plan. In those junctures, the third of the chord is played close to the Pythagorean sharp value, and the sevenths are more equi-tempered, and even wider than those.

IV. 4. The performative context: tempo and dynamics

Intonation is not, however, detached from the other parameters that play a role in performance. As research has shown, fluctuations in tempo and dynamics may influence the ways in which we perceive and approach this parameter[86]. Also, in more general terms, the impression that a performance creates in us is produced by the combination of *all* the performative strategies. As I said elsewhere:

> […] the separation of sound into discrete parameters is artificial, and, thus, parameters typically do not work […] in isolation. In fact, their […] significance generally depends on their combination in specific contexts and to particular purposes, and, thus, there is no predefined hierarchy among them.[87]

And Casals' performance practice is no exception in this respect. In fact, it has already been studied how larger "mistunings" is his and Horszowski's 1936 recording of the first movement from Brahms's F major cello sonata, op. 99, combine with larger asynchronisations between the two

[86] See notes 15 and 20 above.

[87] Ana Llorens, "Creating structure through performance: A re-interpretation of Brahms's cello sonatas" (doctoral thesis, University of Cambridge, 2018), 249, https://www.repository.cam.ac.uk/handle/1810/278796.

instrumental parts and grand-scale tempo fluctuations to produce a sense of tripartite division and expressive tension within the main sections in the sonata structure[88]. As it happens, the omissions in the quote above referred to the formal and structural effects of performative parameters, which, importantly, Casals' intonational decisions in Bach's prelude epitomise too.

Figure 12 represents the fluctuations in tempo and dynamics (in red and green, in the two right-hand y axes), as average values for each harmony in the piece, i.e., mostly every two bars. Coordinated decreases in both of them occur before the resolution into g minor after the scalar passages in bar 62 and, again, although with less intensity, before the return of the opening materials in E flat major in bar 82, thus potentially signaling moments of important structural division[89]. In blue, absolute cent deviations are given, with respect to the theoretical equi-tempered values in scenario ii, i.e., as measured from the preceding note. As for tempo and dynamics, for easier understanding and conjunct visualisation, average values are represented for each harmony in the piece.

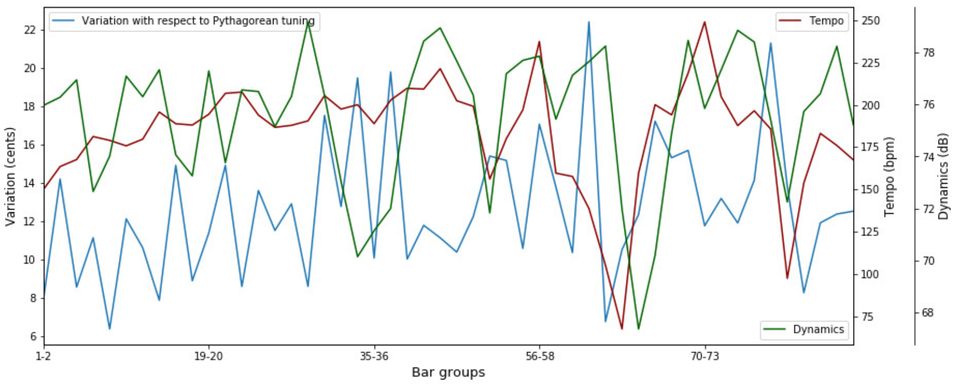


Figure 12. J. S. Bach, Suite for solo cello no. 4, BWV 1010: Prelude, recording by Pau Casals. Intonation (absolute values in scenario ii), tempo, and dynamic fluctuations

[88] *Ibid*., 209-211.

[89] For references on "arch-phrase" theory, see Nicholas Cook, "The Conductor and the theorist. Furtwängler, Schenker and the first movement of Beethoven's Ninth Symphony", in *The Practice of Performance. Studies in Musical Interpretation*, ed. John Rink, 105-125 (Cambridge: Cambridge University Press, 1995); Alf Gabrielsson, "Timing in music performance and its relations to music", in *Generative Processes in Music. The Psychology of Performance, Improvisation, and Composition*, ed. John A. Sloboda, 27-51 (Oxford: Clarendon Press, 1988); Mitchell S. Ohriner, "Grouping hierarchy and trajectories of pacing in performances of Chopin's Mazurkas", *Music Theory Online* 18, no. 1 (2012), http://www.mtosmt.org/issues/mto.12.18.1/mto.12.18.1.ohriner.php; Bruno H. Repp, "Patterns of expressive timing in performances of a Beethoven minuet by nineteen famous pianists", *Journal of the Acoustical Society of America* 88 (1990): 622-641; Neil P. McAngus Todd, "A model of expressive timing in tonal music", *Music Perception* 3 (1985): 33-58; and Neil P. McAngus Todd, "The dynamics of dynamics. A model of musical expression", *Journal of the Acoustical Society of America* 91 (1992): 3540-3550.

Such average intonational deviation tends to be larger than 8 cents except on two occasions where it approaches 6 cents: coinciding with the tempo and dynamics increases, in bs. 62 and 82. Moreover, they follow moments of progressively higher deviation, up to 20-22 cents, in this manner producing a marked increase of expressive distension and resolution which contributes to the sense of structural division discussed above.

Obviating the differences in scale and, hence, representation, in Casals' recording of Bach's piece we can notice two big structural—and tensional—arches. These arches do not spring from the fluctuations in two performative parameters only—tempo and dynamics—, as much literature has explored, but rather from three. Intonation may not be sensed to contribute to the effects of tempo and dynamics, but to ally itself with them on equal footing to produce the sense of ebb and flow that shapes this performance[90].

A similar phenomenon occurs in Casals' recordings of Brahms's movement, where the various parameters coordinate themselves on the small scale to produce concurring effects. Here, they do so too, yet on the large scale to give Bach's prelude a high-order organisation. Within it, the small-scale intonational deviations are not constricted by any pitch-stable instrument, such as the piano in Brahms's: Casals can freely follow and, furthermore, exploit the direction of the melody and the harmonic tensions and resolutions through his intonational practice in a manner that intonation contributes and at the same time transcends the "expressive" role that the cellist himself seems to have assigned to it. Whether or not this was a conscious yet unspoken performance principle of his we cannot know. What we can know it that its effects go beyond any automatic or, perhaps, intentional positioning of the fingers on the fingerboard to become a grand-scale structural factor in the performance.

V. Beyond expressive intonation

Whereas Casals tends to play slightly sharp, the detailed observation of the data, grand-scale statistical conclusions aside, shows that, depending on the specific juncture, he played minor, major, diminished, and augmented intervals, be they seconds, thirds, sixths, or sevenths, both sharp and flat. The same occurs in the case of the perfect fourths, fifths, and octaves: the cent variation with respect to the theoretical values in the equal temperament and the Pythagorean tuning, measured in relation to the fundamental note of the chord or to the immediately preceding note, can occur in both directions.

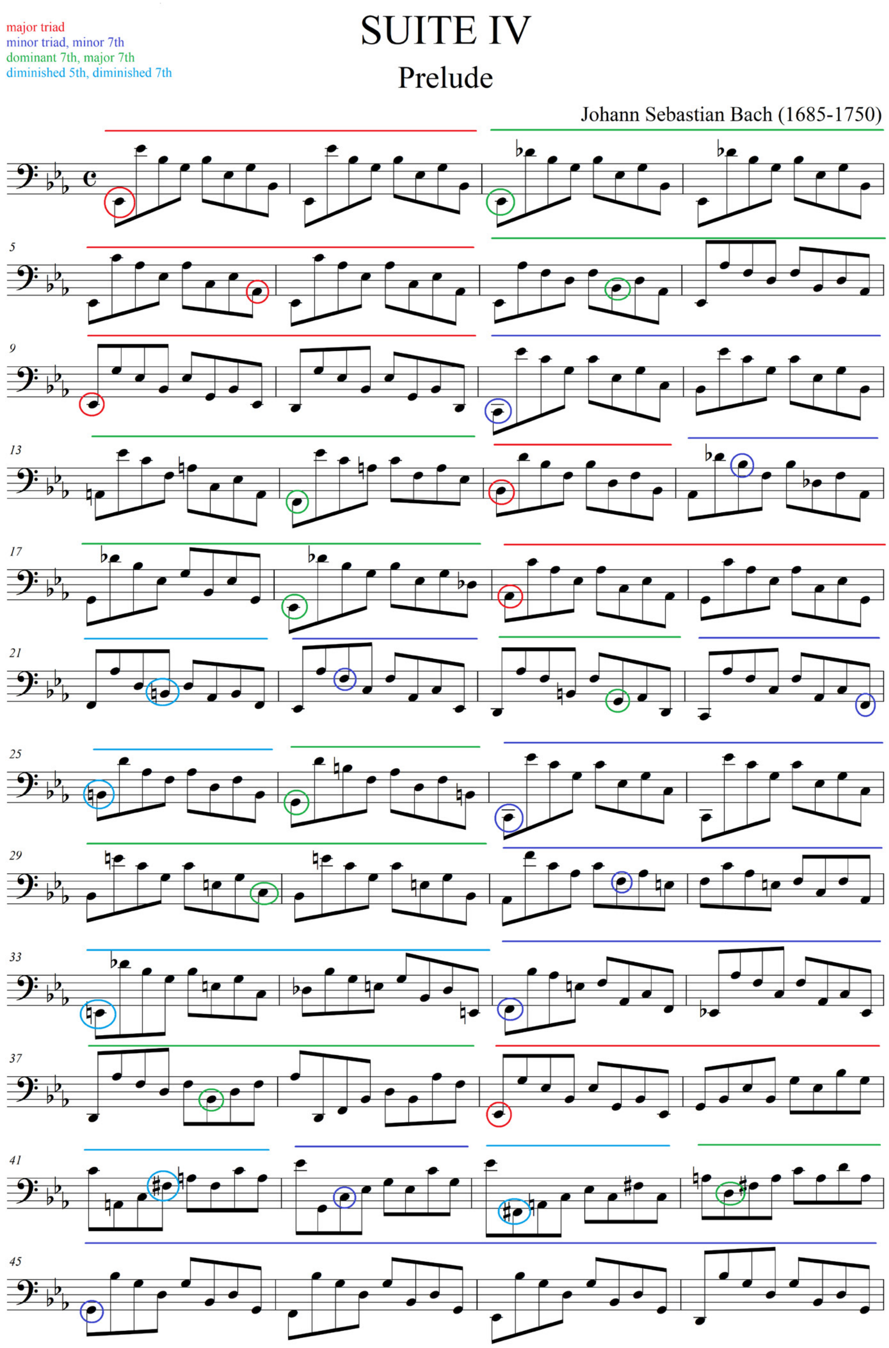
[90] Employing the same expression, John Rink speaks of music's *shape*, as opposed to *structure*, in terms of "momentum, climax, and ebb and flow". See John Rink, "Review of Wallace Berry: *Musical Structure and Performance* (New Haven: Yale University Press, 1989)", *Music Analysis* 9, no. 3 (1990), 323.

However, the deviations occur against a stable harmonic framework. First, whereas in the scalar passages and the "expressive" intervals of the passing notes Casals seems to be thinking melodically, in the harmonic notes of the arpeggios he seeks for a more neutral and stable compromise. In other words, the large deviations in scenario iii respond to Casals' "expressive" intentions towards a close-to-Pythagorean tuning that nonetheless remains more equi-measured in the harmonically framing intervals: unisons, perfect fourths, perfect fifths, and perfect octaves. Within this framework, the cellist tends to play the third degree sharp (as in the large ascending leaps and in the diminished chords), and also avoids narrowed, just sevenths, perhaps looking for higher harmonic brilliance. This may indicate that Casals had a predominantly melodic approach to intonation, in such a way that he would "mistune" certain intervals to attain what he considered a good note-to-note "expressive" performance.

In the melodic passages, in the second place, Casals' follows the fluctuations in the melody: if this goes up, his intonation becomes sharp too, and vice versa. In passages with augmented and diminished intervals, moreover, such deviations contribute to the sense of accumulating intonational tension that resolves into the following harmony. In other words, Casals exploits the compositional play between harmonic tension and resolution inherent in Bach's E flat major Prelude to provide it with the corresponding performative play between intonational tension and resolution too.

His manipulation of intonation in conjunction with tempo and dynamics convert bar 62 into the start of a new large-scale section, the structural power of which is placed, through performance, at the same level as the return to the main harmony twenty bars later, in bar 82. In this manner, the harmonic fluctuation between E flat major and g minor, emphasised through the unstable resolution in bar 45, is disputed in two equal planes, in a metaphor of the *compromise* for two systems that characterises Casals' intonational practice. His is a compromise for the options that allowed him to be more expressive within a more controlled, natural framework of fourths, fifths, and octaves. Similarly, through the largest variations in the performative parameters he emphasised harmonic expressivity, i.e., the contrast between keys, within the framing structure of the E flat major arpeggios that prelude Bach's fourth suite.

Appendix


major triad
minor triad, minor 7th
dominant 7th, major 7th
diminished 5th, diminished 7th
SUITE IV
Prelude
Johann Sebastian Bach (1685-1750)
5
9
13
17
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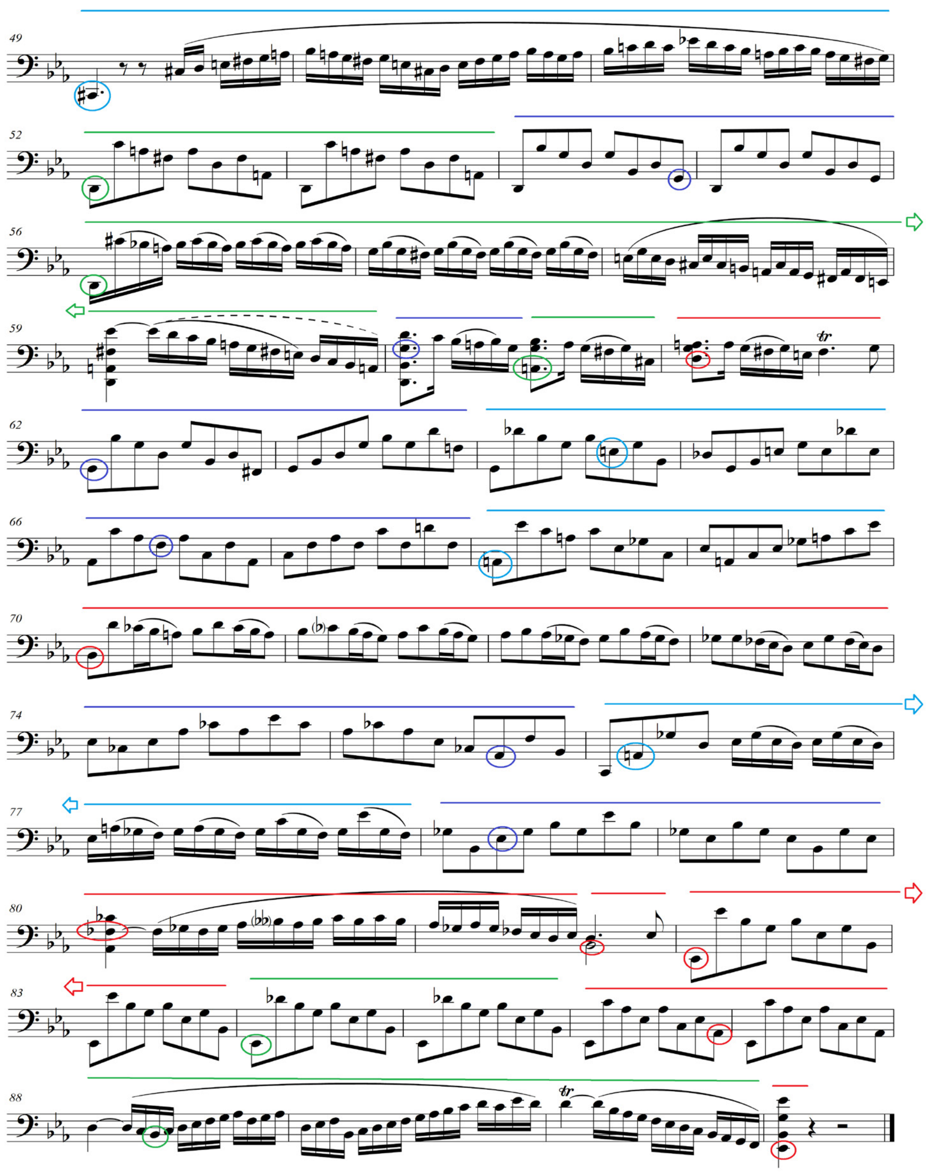
33
37
41
45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