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Designing strategies for Topological Interlocking Assemblies in architecture. Flat Vaults

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Abstract

The modular interlocked blocks in flat structures are known in ancient buildings with pure-compression constructions. Over the last two decades, this structural bond has become relevant, studied by mechanical engineers, and material scientists due to the properties and design freedom that modular structures have. The structural hierarchy existing in topologically interlocked structures enhance the performance, allowing to design and fabricate custom block elements. The main reason to consider this system is that, from the architectural perspective, it is composed by identical modular elements, and it discretizes flat or curved surfaces into elements that work only by contact and compression. This article presents preliminary studies for its application and different approaches for designing discrete interlocked assemblies with a focus on the application for architectural structures: studying the structural performance of contact analysis and introducing the combination of topological interlocking with different structural principles.

Keywords: Topology, interlocking, vaults, patterns, pure-compression, post-tensioned.

1. Introduction

Topological interlocking is a design principle, which elements of a particular shape are arranged in such a way that the whole structure can be held together by a global peripheral constraint" *Dyskin et al.* [1]. Besides being a complex structural system to analyze, it is composed of identical modular elements, providing flexibility during the assembly process, not being necessary any fastener or adhesive, since the contact mechanism of it topology is enough to keep a locked structure. From the traditional architectural perspective, the flat vaults discretize horizontal surfaces into elements that work only in pure compression, likewise the masonry vaults, however transmitting the load not directly by its shape, but it contacts elements geometry. The perimeter nevertheless acquires a critical role, receiving a horizontal pushing pressure from the inside. This paper will introduce a series of alternative designs implementing topological interlocking in architectural scale, fabrication methods, and experimental analysis, laying the groundwork for further mechanical analysis.

2. References review

2.1 Topological Interlocking in Architecture: stereotomy

Bridging the boundary between the knowledge of nature and human ingenuity, it would be placed with realizations such as the flat dome of El Escorial, where Juan de Herrera covered a space of 7.81 meters, as in *Rabasa and López et al.* [2] with a structure of just 28.1 cm (1 foot), having a slenderness of just 1/28. This vault can be considered in the words of *Ávila et al.* [3] as "the limit of an infinite radius dome, which does not have an appreciation of curvature in the section." The success of this vault is in the perfection of the work of its elements, the large size of the pieces (concentric rings of conical faces), and the reduced thickness of the mortar used. Another example of architectural boasting about the construction of flat vaults was proposed by the French engineer *Joseph Abeille* in 1699, proposing a unique piece that would solve the development of this type of flat vaults. According to *Rabasa et al.* [4], the theories of *Abeille,* or the variables proposed by *Freizer* (1737) were carried out in Lugo and at the *Pontón de la Oliva*, in Madrid.

The knowledge obtained is the result in many cases of a process of trial and error from which the calculation models proposed even more audacious solutions. The appearance of new materials and the refinement of calculation and simulation let now to push the limits established in terms of dimensions and slenderness, but the forms continue to fit those already refuted by the experience of centuries.

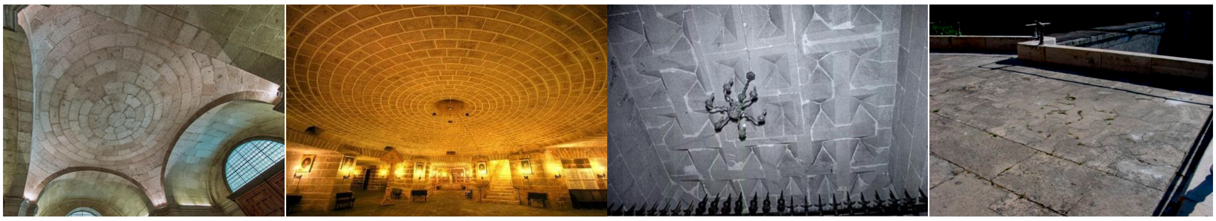


Figure 1: Interlocked vaults examples (from left to right): Monastery of Escorial (1565), Cádiz Cathedral's Crypt (1730), Lugo's Cathedral (1699), Pontón de la Oliva (1853)

2.1.1 Historical patterns

The masters of stereotomy produced and replicated modular blocks that guarantee the uniform load transmission (Figure 2). Therefore, there were several typology patterns, depending on its regularity/irregularity or the circular/orthogonal distribution. In addition to the *Abeille's Bond*, in France, the mathematician *Truchet*, and the engineer *Freizer*, complemented later this vault system with patterns alternatives, *Rabasa et al* [4]. Moreover, they introduced a non-orthogonally (hexagonally) patter to transmit the forces, showing it exists certain flexibility in building interlocking structures.

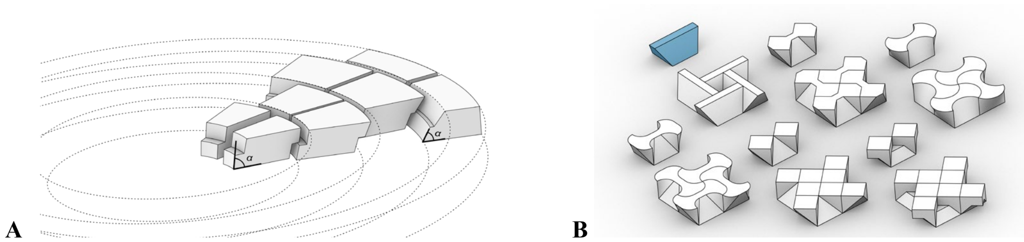


Figure 2: Patterns comparison: (A) Circular vault of Escorial, (B) (blue) *Abeille's Vault*, and *Truchet* and *Freizer* topologies alternatives.

2.3. Advances in Topological Interlocking: analysis and design

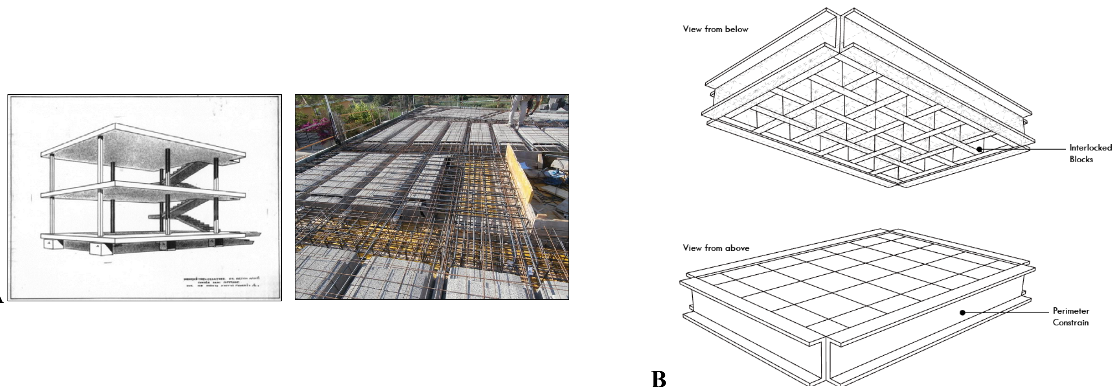
The interest of TI (Topological Interlocking) became relevant in areas such as material science and mechanical engineering, also known as *Architectural Materials*. Clear examples demonstrated the performance with various materials, [1][5][6]. Later studied the systematization of the TI mechanical behavior, [7][8][9]. Many cases, research complemented with experimental tests, that established advantages of TI mechanics with brittle materials, such as glass [10], ceramic [11], or even impact with monolithic concrete [12]. The manufacture of interlocking materials has received a considerable increase in alternatives, thanks to the use of additive manufacturing technologies [13].

2.3.1 Topological interlocking patterns

Pattern studies are usually performed by *Design Experiments* [14]. The catalog of solutions is extensive, even proposing interlocked patterns inspired on organic geometries such as the osteomorphic pattern based on a hexagonal grid [15]. In order to balance the geometry, parametrical investigations [16] can determine the importance of contact surface, analyzing the non-linearity of the results.

3. Design Strategies

Traditional industrial constructions establish hierarchy, with elements of large dimensions and various materials, creating high entropic structures (Figure 3a). TI is based on different hierarchy but composed of identical blocks (Figure 2), making possible creating interlocked packages (Figure 3b).



A

B

Figure 3: Two different typologies of construction: (A) *Maison Dom-Ino*, *Le Corbusier* (1914), variety of elements in concrete slab construction (nowadays), (B) proposal of interlocked package slab.

Interlocked geometries appear in various traditional constructions. Over centuries these solutions remain in good condition since in many cases the primary degradation of constructions comes in the bonding material. Therefore, it is convenient to understand the mechanical properties and the transmission of moments of reciprocal structures.

A reciprocal structure uses its elements to lock them and transmit loads with tension-compression. TI is a three-dimensional reciprocal pattern; the pieces receive moments when some push others. Conventionally, the TI blocks have a moderate size, in order to avoid high deflection of the material (Figure 2), and not to be deformed receiving stress during the transmission of the loads. On the other hand, the geometry of the patterns can be more complex increasing the contact surface with concave-convex patterns as a second interlocking, to prevent the structure failure due to the slips of the elements, tilt of assembly, or the failure of the central element, generating interlocked slabs, [16]. However, the stress concentrates in the tiny stuck out parts, tending to have a fragile failure.

4.1 Preliminary FEM Analysis.

To prove the flat vault configurations, *Discrete Element Method* simulations can be arranged to obtain a static behavior (Figure 6). However, it is required to perform *Finite Element Method* simulations to consider the contact analysis [17], obtaining information about stress, strain, and moment capacities.

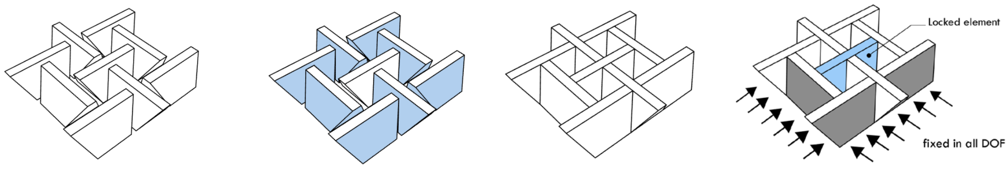


Figure 6: TI *Abeille Pattern*. Assembly, Contact Surfaces, and exterior surfaces fixed in all degrees of freedom.

One of the *Truchet Patterns* is arranged in a 9-element composition (60 x 60 x 13 mm) (Figure 6) to perform a mechanical simulation, with 100 N incremental load applied in the central element. With briefly *Design Experiments*, it can be analyzed the behavior of the interlocking assemblies to redesign various geometries. On the simulation diagrams, the stress tends to be on specific element positions. For materiality, it was chosen conventional concrete without reinforcement. Besides, the simulation is contrasted with a block of identical dimensions, boundary conditions, load position, and material, to compare the maximum displacements [Figure 8].

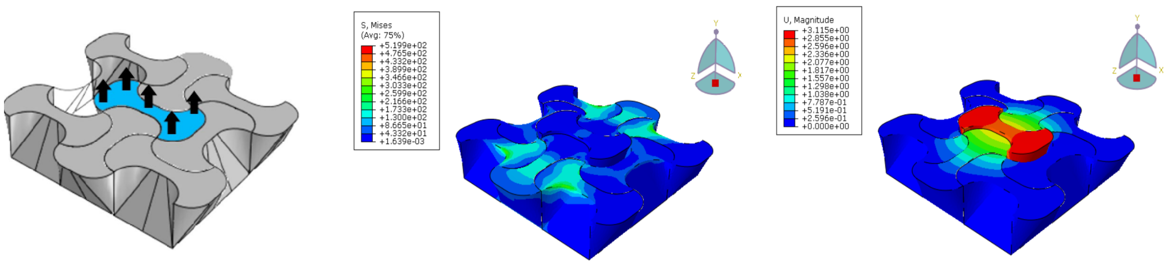


Figure 7: TI FEM simulation with *Truchet Pattern*. Geometry, Stress von-Mises, and Displacement.

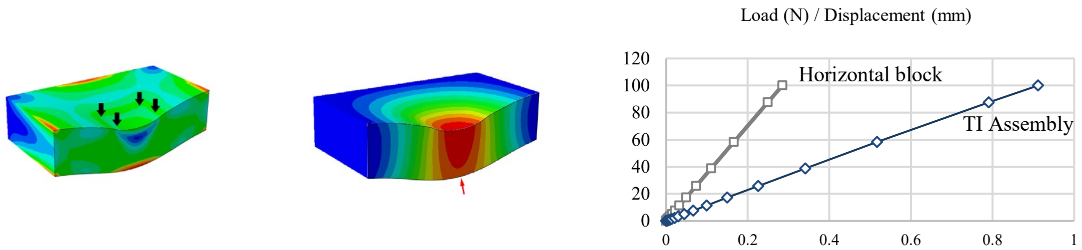


Figure 8: Horizontal block: (from left to right) Stress von-Mises, displacement, and load/displacement graph.

The horizontal block has distributed stress, and the displacement tends to affect the whole, while the TI, as a discrete system, does not damage the material. The displacement graph shows that TI tends to collapse due to geometrical constraints. The traditional construction presented (2.1) demonstrates that TI geometric solutions in building structures perform well with materials in pure compression.

4 Architectural prototypes.

Topologically interlocked assemblies as a substitute for traditional slabs-construction may seem inexperienced, due to the sparse construction practice today. Understanding its mechanics, trial and error prototypes were assembled using the *Truchet patterns* (Figure 9b). Different models were built, testing multiple possibilities to build the perimeters. Finally, it was decided to make double perimeters: a rigid ring, with 4 linear elements that keeps the TI assembly in place (Figure 9a). This paper presents three possible ways to approach it as a continuing and inspiration of historical examples (4.1, 4.2, 4.3).

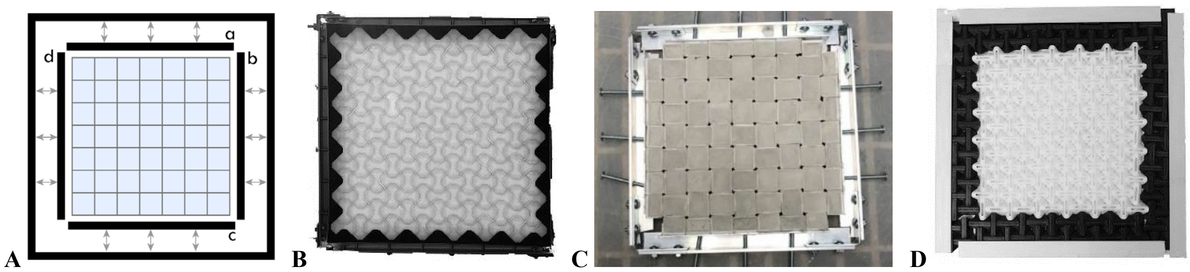


Figure 9: (A) Perimeter composition, 3d printing models as TI slabs: (B) Flat Vault with *Truchet Pattern*; (C) reinterpretation of *Abeille Pattern* (4.1), with interlocked shells as lost formwork (Figure 11), and mortar as stiffener material; (D) Lightweight flat vault alternative (4.2).

4.1 Reinterpretation of Abeille's Vault logic.

Once the basic concepts of IT mechanics were understood, here it is proposed the possibility of using materials that work well in pure compression, but that they can also acquire different forms, such as granular materials like sands or clays, added to previously placed shells, as lost form-work.

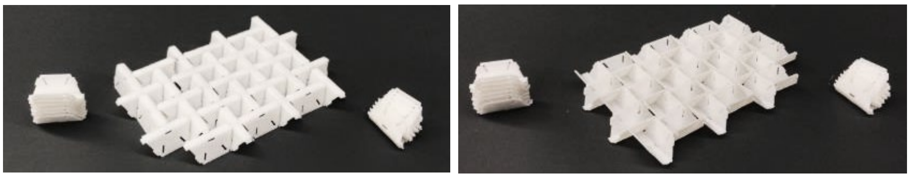


Figure 11: Scaled 3d printing shells with double interlocking design to avoid formwork during assembly.

This modular example has the flexibility of a lightweight structure, later stiffened with infill material. The shells could be manufacturer through forming rigid panels, later processed and positioned on a horizontal surface. Then the double perimeter is built locking the elements, prepared for the stiffener.

With preliminary FEA (Figure 13c) it is noticed the stress does not distribute uniformly, what it is taken into account for the infill material deposition, opting then for a cross filling configuration (Figure 12b).

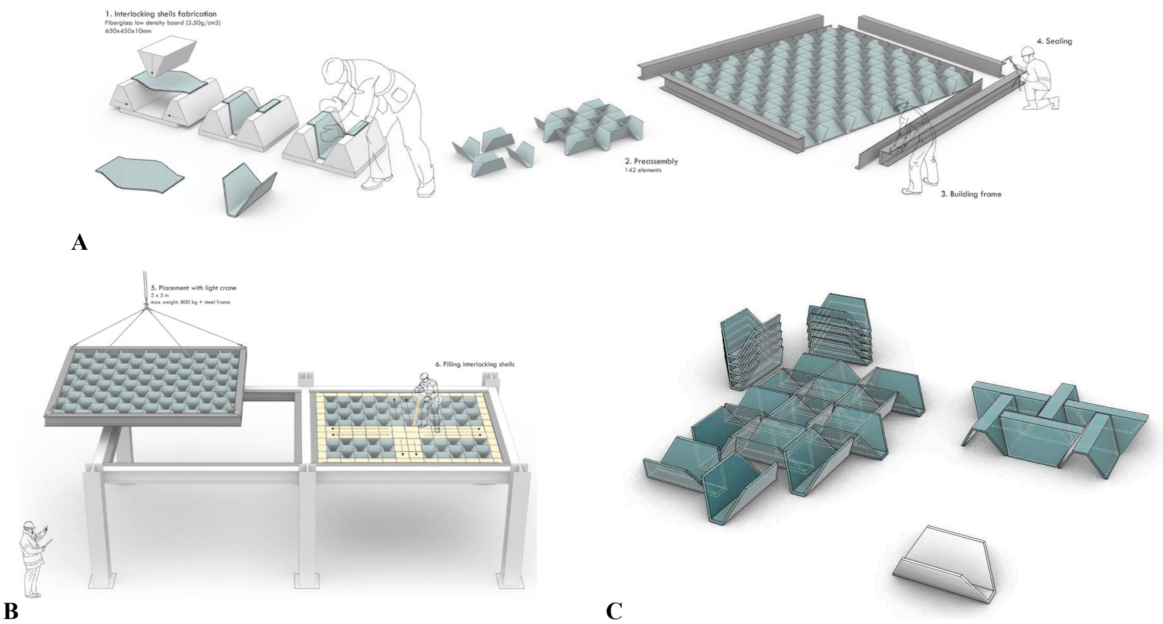


Figure 12: Interlocking shell slab. Assembly. Interlocking shells as lost form-work.

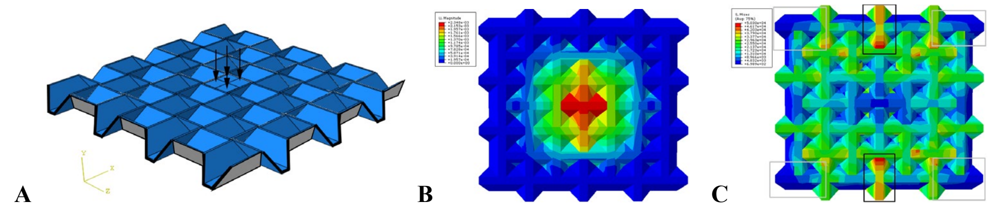


Figure 13: FEA analysis. (13c) Displacement (13d) Stress von Mises.

4.2 Lightweight flat vault alternative

As a reinterpretation of the traditional flat vault bond, the block elements here are not massive, introducing a wireframed-block variation. Additionally, it was introduced the study of the geometrical optimization of individual blocks, reducing their volume and weight, but making them stronger. While reducing its geometry, the contact surface is reduced from the original (Figure 2), that means it would create more significant contact stress in less area, being necessary to reinforce the blocks. This variation provides space in the inside (Figure 14b); that means, the vault can be used to pass through steel rods to hold the perimeter. The experiments with interlocking usually have a static perimeter that works as a traction ring. With this solution, the perimeter ring is replaced with profiles, $a + c$ and $b + d$ (Figure 14a) that maintain the position by the tractioned steel rods that pass through the blocks (Figure 14a).

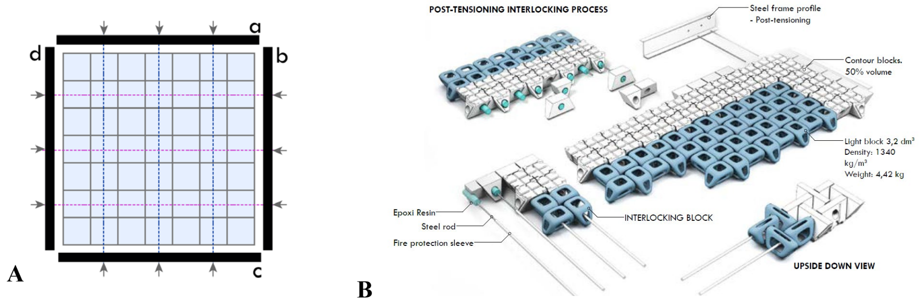


Figure 14: (A) perimeter alternative, (B) composition of the lightweight interlocked vault.

4.2.1 Post-Tensioned Perimeter

Because the principle of this system is the horizontal transmission of forces by contact, it explains the massive stone buttresses in historical constructions (Figure 1. What is proposed here is a post-tensioning system, which consists of the 2 by 2 beams, previously shown, that compress the blocks as a post-tensioning stiffener. Since this architectural proposal would acquire specific flexion and transmission of moments, unlike the traditional stone vault, it can opt for a hybrid system. However, the central elements of the vault can move in a positive or negative direction of Z producing a *bucking behavior*. This property can provide the possibility of generating desired curvatures, i.e., creating positive curvatures, or hybrid *interlocking & masonry vault*, that would tend to work and transmit loads better.

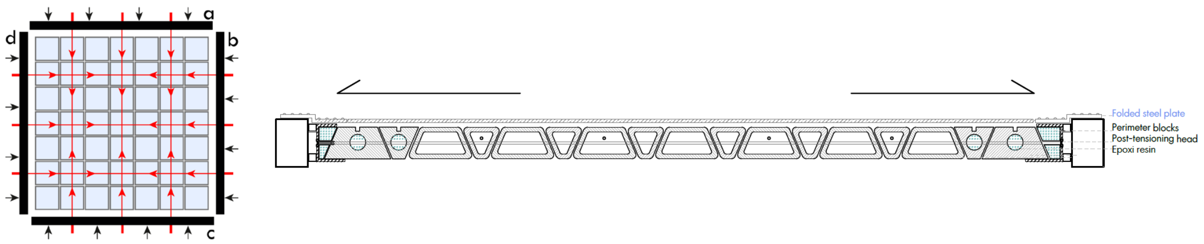


Figure 15: Construction detail of the post-tensioned lightweight interlocked vault.

With FEM simulations, this *bucking behavior* also appears, and after several *Design Experiments* with multiple load cases, it is observed that it comes from a nonuniform post-tensioning process. However, a simulation with a uniform post-tensioning could not converge since the corner elements cannot receive two directional loads at the same time, but, with the experience obtained, a correct post-tensioning would produce a neutral curvature in the center.

To conclude, surfaces can be generated with precise curvatures if the tension of both sides is different, taking advantage of the design of the vault, but complicating the analysis. An example of a robotically assembly is proposed [18], as a fully automated method to be applied in the architectural industry.

4.3 Pattern alternative

Hierarchy studies demonstrated that the geometry is crucial in TI, i.e., *osteomorphic* pattern. Here another reciprocal pattern variant is proposed based on the regular subdivisions with the *Cairo Tiling*, with two layers hexagonal-rhombus pattern (one rotated 90º). In this way, ruled surfaces connect both patterns, arranging up to 6 blocks in contact to distribute the loads (Figure 17). This contact geometry came as inspiration by the recently discovered subdivision packing of epithelia, known as *Scutoids*.

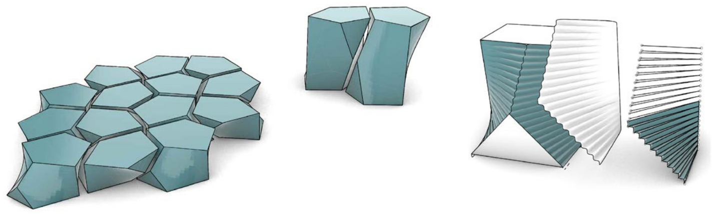


Figure 17: Pattern alternative based in the Cairo Tiling, creating a ruled surface to connect two pentagons.

With further investigations, these pentagons were deformed, in rectangles 1: 2, which likewise, both 90º patterns and contact surfaces generated graphically through an optimized vertex approximation (Figure 18). Finally, it is also studied the possibility of transforming the ruled surfaces that join the patterns in a double interlocking (Figure 19a). With 3d printing models, it was possible to prove how these patterns have bending capacities that allow the positioning of different pieces without formwork (figure 19b).

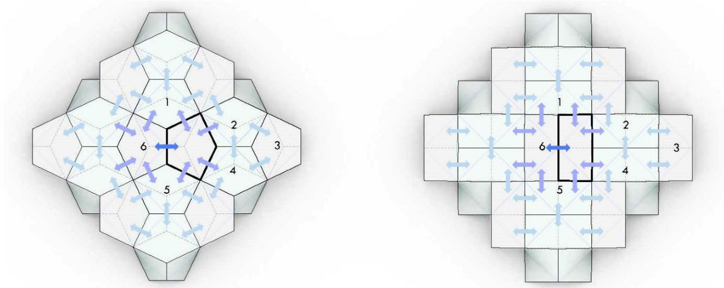
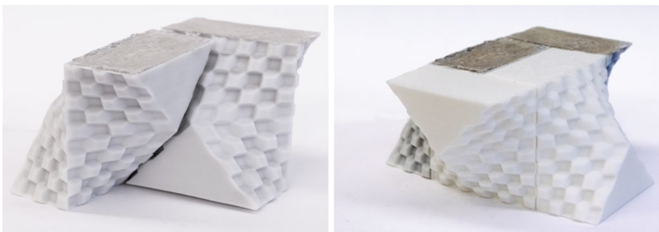


Figure 18: Pattern alternative based in the Cairo Tiling, creating a ruled surface to connect two pentagons.

A 

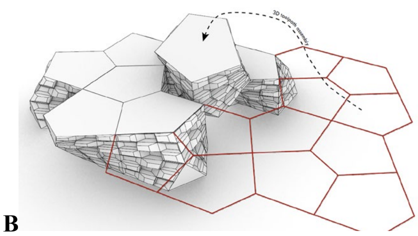
B 

Figure 19: (A) Contact surface incrementation. (B) 3d toolpath for precise automated assembly of TI.

Conclusions

The prototypes presented here establish three main areas for further research of TI in architecture: how molecular materials can be applied; the systematization: possible alternatives as tensioned perimeters and post-tensioning; and the study of geometric pattern alternatives. The discoveries obtained from mechanical engineering can be extrapolated at the highest scales and analyze individual blocks to reduce material, and possible further investigations, with design experiments and statistical optimizations.

The use of this mechanics intelligence can also generate architectural solutions more sustainable than traditional ones: potential local and less disordered materials; and customize interlocked blocks with precise manufacturing techniques.

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