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Real and apparent direction of inertia in the ultimate limit state in doubly symmetrical reinforced concrete sections

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ABSTRACT

The principal direction of inertia in the ultimate limit state under axial load and biaxial bending of a doubly symmetrical reinforced concrete section is not the same as the direction

of the principal axis of symmetry. If a hyperbolic stress–strain relationship is used to describe the behavior of concrete in compression, then, to some extent, the maximum capacity direction deviates from the apparent main axis of inertia (the main axis of symmetry). This study explores the real maximum capacity direction of bending of two reinforced concrete sections with a variable amount of steel using two different axial compression loads and two different stress–strain relationships for concrete (parabolic-rectangular and hyperbolic). The results are presented in a collection of interaction diagrams.

KEYWORDS: Columns, concrete structures, stress analysis.

INTRODUCTION

The numerical difficulty involved in the exact integration of concrete and steel stress on a reinforced concrete section subjected to axial load and biaxial bending has led to the adoption of simplified constitutive models for stress–strain relationship of concrete in compression in regulatory codes. In the ultimate limit state, ACI-318-14 (2014) allows replacing the real stress response profile of the compressed concrete area by a rectangular block with reduced depth and a constant stress value (this hypothesis was formulated and developed by Withney 1956). Alternatively, a non-linear analysis may be performed in which equilibrium conditions as well as strain compatibility are considered. For this analysis, it is necessary to know the stress–strain relationship for concrete and steel. Parabolic-rectangular models (Fig. 1a), in which the compressive stress is achieved for the ultimate strain value, are commonly used.

A more realistic stress–strain relationship for concrete, which is consistent with, e.g., Hognestad 1951, Kent 1971, and Sargin 1971, is shown in Fig. 1b. The stress due to the ultimate strain is not the maximum compressive stress of concrete. In these models, there is a significant decline in the stress compared to the maximum stress for strains approaching the maximum value of the material.

The choice of the stress–strain relationship for determining section capacity is a relevant factor in the resolution of the single axial and biaxial load compression issues. If a biaxial interaction diagram $\left(M_{x,}M_y\right)$ is calculated for a section with two axes of symmetry and a constant axial compression load with simplified stress–strain relationships for concrete (Fig. 1a), the hypothesis by Morán (1972) with regard to the convexity of these diagrams appears to be confirmed. In this situation, the maximum capacity value is attained in the direction of the main axis of symmetry of the section (the x axis), which is also the main axis of inertia in the ultimate limit state.

In situations with high axial compression loads and hyperbolic stress-strain relationships (Fig. 1b), the maximum bending capacity combined with axial compression load need not be in the direction of the apparent main axis of inertia (the principal axis of symmetry), but it may deviate to some extent. This implies that, in some cases, the interaction biaxial diagrams may have concavities in the areas near the symmetrical directions of the section, that is, the apparent main axes of inertia $(0°, 90°)$. Regarding symmetrical sections, under axial load and single bending, the capacity may be overestimated as well, but the direction where the maximum capacity is obtained is perpendicular to the external moment direction, as expected.

Published interaction diagrams (Fig. 2), where a simplified concrete constitutive model was used, as well as typical calculation models based on uniaxial equivalent eccentricity (Pannell 1959, Bresler 1960) do not reflect the actual loss of the section capacity on the symmetry axes $(0°, 90°)$ owing to the use of a simplified stress–strain relationship. For these directions, section capcity is overestimated, and therefore unsafe situations may arise during the design stage.

Accordingly, this study aims to determine, clarify, and complete related knowledge in the theory of structures, as well as to evaluate the possible implications for the design of reinforced concrete sections under compression loads and biaxial bending. It also investigates the rotation between the principal axis of symmetry and maximum inertia in the ultimate limit state. Moreover, even though interaction diagrams have been extensively studied, the previously mentioned numerical singularity is novel and has never been deeply explored.

SIMPLIFIED STRESS–STRAIN RELATIONSHIP FOR CONCRETE IN AXIAL LOAD AND BIAXIAL BENDING

To design reinforced concrete sections subjected to an arbitrary normal state of strength (axial load and biaxial bending), a widely adopted an extensively used technique is to replace the two eccentricities on the section axes by a single eccentricity on the main axis, which leads to a state of external strength equivalent to the original. From a numerical point of view, the design in the context of compression and single bending is simpler than the general problem of axial load and biaxial bending. In the former, the direction of the neutral axis is known and is the same as the perpendicular direction of external eccentricity. In the latter, the direction of the neutral axis is unknown.

Pannell 1963 established a geometric model to determine the interaction diagrams for square sections with a homogeneous distribution of reinforcement on the four faces (section with double symmetry), and used two fundamental hypotheses as a starting point (the first of which will be shown not to be necessarily true):

1. The maximum capacity values are on the planes of symmetry of the section, that is, the main directions $(0^\circ, 90^\circ)$.
2. The minimum capacity value is on the plane of the diagonal of the section, bisecting the main directions (45°).

Furthermore, assuming that the interaction diagram must be a continuous curve that is derivable at all points, a model was formulated that could determine the values of biaxial failure moments (M_x, M_y) for arbitrary directions, in which a curve is described that contains the three known points (nominal moments of the section capacity according to the axes of symmetry, and the direction of the diagonal), as shown in Fig. 3.

In a method by Bresler 1960, the axial load (P_i) leading to section failure for an arbitrary eccentricity direction is linearly inferred. For that purpose, P_i should be determined for each component of the design eccentricity (e_x, e_y), and the maximum axial load without any eccentricity (P_0). This model can be expressed as follows:

$$\frac{1}{P_i} = \frac{1}{P_x} + \frac{1}{P_y} - \frac{1}{P_0} \qquad [1]$$

According to ACI-318-14, this model is a valid strategy for designing sections subjected to compression and biaxial bending.

Interaction diagrams for axial and biaxial bending $(M_{x,}M_y)$ have been obtained for various amounts of steel, transversal section shapes, and axial load levels, for instance, in Parme 1966, Weber 1966, Row & Paulay 1973, Grasser 1981, and Calavera 2008.

All these calculation methods use a simplified stress-strain relationship to describe the behavior of compressed concrete (parabolic-rectangular, Withney's hypothesis). Given this simplification, the depth of the neutral axis is reduced, which implies that the center of gravity of the resultant of the concrete is displaced toward the upper fibers of the section. The mechanical arm is increased, and consequently the values of section capacity (M_x, M_y) are higher than the values calculated for the same section when a hyperbolic stress–strain relationship for concrete is considered. This leads to the appearance of two concavities in the areas near the directions marked by the main axes of symmetry of the section, which have never been considered in published interaction diagrams (Fig. 2) that are currently used for design purposes, such as the diagrams by Montoya 2001 and Calavera 2008.

MAXIMUM CAPACITY DIRECTION OF A REINFORCED CONCRETE SECTION WITH DOUBLE SYMMETRY

To obtain an interaction diagram representing the components of the moment in the ultimate limit state for the two axes of symmetry of the section, a strain plane should be established for each possible rotation angle of the neutral axis (0º–90º if the section has two axes of symmetry). The curvature of the section and the depth of the neutral axis for the failure moment must be determined by imposing the ultimate strain of the extreme compression fiber of the concrete (ε_{cu}) according to ACI-318-14. For each direction of the neutral axis chosen, it may be assumed that there is only one failure plane, and the following equilibrium equations can be formulated with respect to an arbitrary reference system:

$$P = \int_A \sigma_c dA + \sum_{j=1}^{m} A_{s,j}\sigma_{s,j} \qquad [2]$$

$$M_x = \int_A \sigma_c y dA + \sum_{j=1}^{m} A_{s,j}\sigma_{s,j}\, y_{s,j} \quad [3]$$

$$M_y = \int_A \sigma_c x dA + \sum_{j=1}^{m} A_{s,j}\sigma_{s,j}\, x_{s,j} \quad [4]$$

The choice of the stress–strain relationship for concrete in the resolution of the equilibrium equations [2], [3], and [4] determines the value for the section capacity (P, M_x, M_y). In fact, as shown in this study, when it is expected that the maximum capacity is attained on the main axis of symmetry (the x axis), the assumption of a hyperbolic model rather than a parabolic-rectangular model implies that this value is located in another direction. This question is addressed in this study, and a numerical simulation was conducted, in which the interaction diagrams resulting from the use of two stress–strain relationships (parabolic-rectangular and hyperbolic) were compared for two doubly symmetrical sections. To this end, a total of 24 diagrams were obtained, in which three reinforcement amounts and two different axial load levels of compression were considered.

Each diagram in this study was obtained for two different stress–strain relationships for concrete, as shown in Figs. 4a and 4b. The stress–strain curve in Fig. 4b was derived from the equation used by Farah and Huggins 1969, and it is described in polynomial form in Equation [5]; Fig. 4a shows the curve in Fig. 4b with a constant stress value from $\varepsilon_0 = 0.002$ up to $\varepsilon_{cu} = 0.004$ (failure value).

$$\sigma_c = f_{c'}\left[k_1\varepsilon \quad + k_2\varepsilon^2 + k_3\varepsilon^3 + k_4\varepsilon^4\right] \quad [5]$$

Where the constants denoted by k take the following values:

$$k_1 = 0.985 \cdot 10^3$$

$$k_2 = -0.312 \cdot 10^6$$

$$k_3 = 0.306 \cdot 10^8$$

$$k_4 = -0.257 \cdot 10^9$$

The stress–strain curve used to characterize steel was also taken from Farah and Huggins 1969 (Fig. 5), as well as the following polynomial expression [6], which describes it continuously for the entire range of strain:

$$\sigma_s = \frac{f_y}{2}\left(\sqrt{\left(\frac{\varepsilon}{\varepsilon_y}+1\right)^2} - \sqrt{\left(\frac{\varepsilon}{\varepsilon_y}-1\right)^2}\right) \qquad [6]$$

The following characteristic values for concrete and steel were chosen for the diagrams:

$$f_c = 30\ MPa$$

$$f_y = 400\ MPa$$

The resulting diagrams were obtained based on two different rectangular cross-sections with side ratios $h/b = 1$ and $h/b = 2$. Three longitudinal reinforcement ratios were studied for each section. They are defined in Equation ([7], with values $\omega_1 = 0.30$, $\omega_2 = 0.40$, and $\omega_3 = 0.50$; moreover, two compression axial loads are defined according to Equation ([8], with values $\upsilon_1 = 0.85$ and $\upsilon_2 = 0.95$. A mechanical cover with a value of $r = 0.10\,b$ considered in all cases. The two stress–strain relationships for concrete described in Figs. 4a and 4b are used.

$$\omega = \frac{A_s f_y}{bhf_c} \qquad [7]$$

$$\upsilon = \frac{P}{bhf_c} \qquad [8]$$

The dimensionless moments μ_x and μ_y in the diagrams are defined as follows:

$$\mu_x = \frac{M_x}{bh^2 f_c} \quad [9]$$

$$\mu_y = \frac{M_y}{hb^2 f_c} \quad [10]$$

The reinforcement was assumed to be distributed on the perimeter of the section, and a total of 36 elements of equal area were considered to integrate the stress. Paulay 1973 adopted a similar reinforce distribution, and in the present study, it is used because it provides greater generality compared to punctual bars as in real column reinforcement distributions.

The section was divided into a total of 625 elements of equal area, distributed according to a 25 × 25 matrix to simulate the concrete. To validate the analysis, diagrams with three different element size were obtained to perform integration in the compressed block. Thus, the cases 50 × 50 and 100 × 100 (2500 and 10000 elements, respectively) were analyzed as well, and there were no differences between these results and those presented here. This is because precision reduction (with respect to the 10000-element case) occurs in both families of interaction diagrams (for hyperbolic and parabolic-rectangular stress-strain relationship for concrete), and the relative difference is constant; however, the computation time is increased more than fifteen times in the 10000-element case.

A total of 181 points in the rotation range around the neutral axis (from 0° to −90°) were calculated for each interaction curve, as shown in Fig. 6. This is equivalent to obtaining a series of the ultimate limit state planes with the lines (neutral axis) at two consecutive points differing by 0.50º.

It is possible to express Equations [2], [3], and [4] in terms of the sums resulting from each element (concrete and steel) in the cross-section, as follows:

$$P = \sum_{i=1}^{n} A_{c,i}\sigma_{c,i} + \sum_{j=1}^{m} A_{s,j}\sigma_{s,j} \quad [11]$$

$$M_x = \sum_{i=1}^{n} A_{c,i}\sigma_{c,i,}y_{c,i} + \sum_{j=1}^{m} A_{s,j}\sigma_{s,j}y_{s,j} \quad [12]$$

$$M_y = \sum_{i=1}^{n} A_{c,i}\sigma_{c,i,}x_{c,i} + \sum_{j=1}^{m} A_{s,j}\sigma_{s,j}x_{s,j} \quad [13]$$

To avoid assigning a non-real capacity to the section, the stress of each steel element was modified to simulate the displaced concrete area by reducing its value according to the following expression for compressed steel elements [14].

$$\sigma_{s,i} = \frac{f_y}{2}\left(\sqrt{\left(\frac{\varepsilon}{\varepsilon_y}+1\right)^2} - \sqrt{\left(\frac{\varepsilon}{\varepsilon_y}-1\right)^2}\right) - f_{c'}\left[k_1\varepsilon \quad + k_2\varepsilon^2 + k_3\varepsilon^3 + k_4\varepsilon^4\right] \quad [14]$$

Initially, each point in the diagrams was calculated imposing four conditions:

1. The direction of the neutral axis is established (α).
2. The ultimate limit of the section must be achieved by compression in the extreme fiber, which ensures unlimited steel ductility. From an operational point of view, the strain value must be defined to describe the stress–strain relationship, and this was set to $\varepsilon_{yu} = 0.020$, (Fig. 5). This value was not reached in any plane of the ultimate limit state in this study. This ultimate limit model is used in ACI-318.

3. No reduction factors in the materials were used.
4. No reduction factor in the section capacity was used.

The unknown variable of the ultimate limit plane to be determined for each point in the diagram is the depth of the neutral axis. This is obtained iteratively. The process of finding each depth end when the internal axial load (after Equations [11], [12], and [13) have been solved for the postulated plane) is close to the exterior axial load. In this study, the depth of each neutral axis considered valid when the difference between axial loads (sought and calculated) is less than 0.01%, that is, [15].

$$P_d - P \leq 0.0001\,P \qquad [15]$$

Fig. 6 shows the 24 interaction diagrams calculated for the situations described. In every chart, four interaction diagrams are presented, and for every pair of lines with the same axial load, it is possible to observe the different capacity achieved in the symmetry direction of the section, depending on the stress–strain relationship for concrete (continuous line for the hyperbolic and dashed line for the parabola-rectangular).

The maximum value of the capacity moment of the section, as a vector composition with respect to the axes of symmetry according to Equation ([16] (in dimensionless terms) was determined for the 24 cases studied, as well as the rotation angle for which that maximum value was obtained. Effectively, for all the sections in which a parabolic-rectangular stress–strain relationship was used, the maximum capacity value was found for the direction of the axis of symmetry with the greatest inertia, that is, the *x*-axis.

$$M = \sqrt{M_x^2 + M_y^2} \qquad [16]$$

DISCUSSION

Table 1 shows the relevant values calculated for the 24 interaction diagrams shown in Fig. 6. The table headers are explained as follows:

$\boldsymbol{h/b}$**:** Ratio between the sides of the transversal section.

$\boldsymbol{\omega}$: Ratio of steel according to [7].

Stress–strain relationship: type of model describing the behavior of concrete in compression (parabolic-rectangular, hyperbolic).

$\boldsymbol{\nu}$: Axial dimensionless load according to [8].

$\boldsymbol{\mu(\alpha = 0°)}$**:** Dimensionless resistant moment of the section as a vector composition with respect to the *x*-axis of symmetry.

$\boldsymbol{\mu(\alpha = 90°)}$**:** Dimensionless resistant moment of the section as a vector composition with respect to the *y*-axis of symmetry.

$\emptyset$**:** Maximum capacity angle of the section with respect to the main axis of symmetry of the section.

$\boldsymbol{\mu(\alpha = \emptyset)}$**:** Dimensionless resistant moment of the section as a vector composition with respect to $\emptyset$ direction.

$\boldsymbol{\Delta\mu_x(\%)}$**:** Percentage difference between the resistant moment of the section with respect to the *x* axis for the same section considering two different stress-strain relationships describing the behavior of concrete in compression.

According to the values shown in Table 1, for the sections in which a hyperbolic strain–stress relationship is applied, the maximum capacity value is not the same as the direction of the main axis of symmetry, and in all cases the value of the resultant of the maximum moment with respect to the main axis of symmetry (the x-axis) is smaller than the value of the resultant ultimate moment on the same alignment for a parabolic-rectangular stress–strain relationship.

This is because in situations with high axial loads and bending, the maximum stress in the cross section is not located in the top fiber. In this position, the strain is ε_{cu}, and this causes the resultant barycentre of the compressed block of concrete to approach the position of the neutral fibre, decreasing the mechanical arm and reducing the resultant moment.

Fig. 7 shows the stress profile on the compressed area of the analyzed section $h/b = 1$, $\omega = 0.30$, and $\nu = 0.95$, for parabolic-rectangular and hyperbolic stress–strain relationships for concrete, and for a 0.5 × 0.5 m section size. In this situation, the difference between the maximum resistant moment of the section and the resistant moment found for the principal symmetry axis is the highest (9.615%). It can be seen that the mechanical arm of the section in Fig. 7b is lower than in Fig. 7a. It can also be observed that in the position of maximum strain (top fiber in Fig. 7b), the stress decreases with respect to the maximum stress in concrete, which is compatible with the stress–strain relationship shown in Fig. 4b.

CONCLUSIONS

1. In reinforced concrete sections with two axes of symmetry, it cannot be assumed that the ultimate limit interaction diagram is convex in its entirety. This is true at least, for stress–strain relationships for concrete in which the ultimate strain has associated stress values lower than the maximum values.

2. The principal axis of inertia of a rectangular or square reinforced concrete section need not necessarily be the main axis of symmetry of the section in the ultimate limit state, and consequently, the maximum capacity value of the section need not necessarily lie in the expected direction (the *x*-axis).

3. The rotation deviation between the axis of maximum capacity and the main axis of symmetry of a double reinforced symmetrical concrete section increases as the axial load increases.

4. The divergence in the capacity of the section for axial load and single axial bending with regard to the main axis of symmetry of the section for the parabolic-rectangular and hyperbolic models increases as the amount of reinforcement decreases. That is, it may be assumed that for low steel ratios, the reduction in the capacity of the section under axial load and uniaxial bending increases with regard to the main axis of symmetry of the section.

5. The rotation deviation between the axis of maximum capacity and the principal axis of symmetry of the reinforced concrete section increases as the variation between the maximum stress of concrete in compression and the stress for the ultimate strain is increased.

6. For both ratios h/b analyzed in this study, the maximum deviation for the principal axis of inertia from the principal symmetry axis of the section in the ultimate limit state has been found for the square section ($h/b = 1$).

7. The use of a simplified stress–strain relationship to describe the behavior of concrete in compression for double symmetrical reinforced concrete sections and elevated axial compression loads results in overestimation of the section capacity in bending with respect to the symmetry axis.

8. The convexity hypothesis (Morán 1972) cannot be ruled out in interaction diagrams of reinforced concrete sections with double symmetry for stress–strain relationships in which the maximum stress occurs for the ultimate strain. That is, it is not possible to assert that the principal axes of inertia in the ultimate limit state coincide with the axes of symmetry of a section when hyperbolic stress–strain relationships are used to describe the behavior of concrete in compression.

NOTATION

The following symbols are used in this paper:

b = Cross-section width;

f_y = Specified tensile strength of steel reinforcement;

f_c = Specified compressive strength of concrete;

h = Overall height of cross-section;

k_i = Coefficients in the description of the stress–strain relationship for concrete;

r = Mechanical cover of the section reinforcement;

A = Gross section area;

M = Moment resulting from the vectorial composition of its components;

M_x = Moment relative to the *x* axis of the section;

M_y = Moment relative to the *y* axis of the section;

ΔM_x = Difference of the component related to the x axis of the section of the bending moment.

P = Axial load;

P_0 = Nominal axial strength at zero eccentricity;

P_d = Design axial load;

P_i = Nominal axial strength in the ultimate limit state applied at a point $i(x, y)$;

P_x = Nominal axial strength in the ultimate limit state applied at a point $i(x, 0)$;

P_y = Nominal Axial strength in the ultimate limit state applied at a point $i(0, y)$;

α = Neutral axis angle direction;

ε = Strain;

ε_{c0} = Strain of concrete at maximum stress;

ε_{cu} = Strain at which the failure in compression in the concrete is reached;

ε_y = Yield strain of steel reinforcement;

ε_{yu} = Tensile strain of steel reinforcement;

μ = Dimensionless bending moment;

ν = Dimensionless axial load;

σ_c = Stress in a concrete element;

σ_s = Stress in a steel element;

ω = Ratio of reinforcement of the cross-section;

$\emptyset$ = Maximum capacity angle of the section with respect to the main axis of symmetry of the section;

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- **Fig. 1a**. Stress–strain relationship diagram for a parabolic-rectangular model.
- **Fig. 1b**. Stress–strain relationship diagram for a hyperbolic model.
- **Fig. 2**. Interaction diagrams for axial load and biaxial bending for the analysis of reinforced concrete sections (Calavera 2008).
- **Fig. 3.** Ultimate limit area for a pair of moments $M_x - M_y$ and the surface of revolution for the major axis of inertia of the section (Pannell 1963).
- **Figs. 4a & 4b.** Modified and original stress - strain relationship for concrete used by Farah and Huggins 1969.
- **Fig. 5.** Stress–strain relationship used by Farah and Huggins 1969 for reinforcing steel.
- **Fig. 6.** Interaction dimensionless diagrams for sections described in Table 1. The continuous line denotes hyperbolic stress–strain relationship. The dashed line denotes parabolic-rectangular stress–strain relationship.
- **Fig. 7a.** Stress profile in concrete for the analyzed section $h/b = 1$, $\omega = 0.30$, $\nu = 0.95$, for parabolic-rectangular strain–stress relationship, 0.5 × 1 m size member.
- **Fig. 7b.** Stress profile in concrete for the analyzed section $h/b = 1$, $\omega = 0.30$, $\nu = 0.95$, for hyperbolic strain stress relationship, 0.5 × 1 m size member.

Table 1. Geometrical and mechanical definition of the analyzed sections and relevant results obtained from the calculated interaction dimensionless diagrams shown in Fig. 6.

h/b	ω	**Stress–strain relationship**	ν	μ $(\alpha = 0°)$	μ $(\alpha = 90°)$	$\emptyset$ (°)	μ $(\alpha = \emptyset)$	$\Delta\mu_x$ (%)
1	0.30	Parab.-Rect.	0.85	0.133	0.133	0.00	0.133	6.767
1	0.30	Hyperbolic	0.85	0.124	0.124	13.00	0.126	
1	0.30	Parab.-Rect.	0.95	0.104	0.104	0.00	0.104	9.615
1	0.30	Hyperbolic	0.95	0.094	0.094	40.00	0.100	
1	0.40	Parab.-Rect.	0.85	0.159	0.159	0.00	0.159	4.403
1	0.40	Hyperbolic	0.85	0.152	0.152	10.00	0.153	
1	0.40	Parab.-Rect.	0.95	0.134	0.134	0.00	0.134	6.716
1	0.40	Hyperbolic	0.95	0.125	0.125	16.50	0.127	
1	0.50	Parab.-Rect.	0.85	0.186	0.186	0.00	0.186	3.763
1	0.50	Hyperbolic	0.85	0.179	0.179	3.00	0.179	
1	0.50	Parab.-Rect.	0.95	0.161	0.161	0.00	0.161	4.969
1	0.50	Hyperbolic	0.95	0.153	0.153	8.50	0.155	
2	0.30	Parab.-Rect.	0.85	0.139	0.133	0.00	0.139	5.755
2	0.30	Hyperbolic	0.85	0.131	0.124	19.00	0.133	
2	0.30	Parab.-Rect.	0.95	0.109	0.104	0.00	0.109	9.174
2	0.30	Hyperbolic	0.95	0.099	0.094	50.00	0.104	
2	0.40	Parab.-Rect.	0.85	0.170	0.159	0.00	0.170	4.706
2	0.40	Hyperbolic	0.85	0.162	0.152	13.50	0.163	
2	0.40	Parab.-Rect.	0.95	0.141	0.134	0.00	0.141	6.383
2	0.40	Hyperbolic	0.95	0.132	0.125	21.50	0.134	
2	0.50	Parab.-Rect.	0.85	0.200	0.186	0.00	0.200	4.000
2	0.50	Hyperbolic	0.85	0.192	0.179	10.00	0.193	
2	0.50	Parab.-Rect.	0.95	0.173	0.161	0.00	0.173	5.202

2	0.50	Hyperbolic	0.95	0.164	0.153	16.50	0.165