

## SIGNIFICANCE OF FEATURES IN A COMPARISON OF RESULTS FROM STOCHASTIC AND NON-STOCHASTIC METHODS: THE CASE OF SPANISH RETAILING POST REGULATE PROCESS

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We compare the results of the scores of efficiency by means of the methods DEA and SFA for a sample of firms of the sector Spanish retailer. The evaluation by means of Stochastic kernel and Sizer tool shows that the results are robust in the five used models. Indicating a loss of efficiency of the firms in the time.

*Keywords:* Efficiency, Sizer tool, Stochastic kernel, Retailing, Regulation, DEA, SFA.

*JEL:* L5, C5, C6

### 1. Introduction

Efficiency scores are determined for Spanish firms in Retail industry using both parametric and non parametric methods. The objective of this work is to determine the probability of changing from any level of efficiency in the initial year 1996 (beginning of the regulation process<sup>1</sup>) at any level in 2002. Therefore the position is not to make comparisons of the analysis of the ranking of efficiency through the different methods. In particular we are interested in analyzing the robustness of the obtained results of the different *opposite* models through two evaluation methods; stochastic kernel and the tool Sizer see Godtliebsen *et al.*, (1999). An interesting contribution of this work is related with the dynamic analysis of the efficiency and its comparison with the different methods as well as the techniques employees to carry out this comparison.

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Both DEA and SF analysis are popular methods for assessing relative efficiency. Unfortunately, there is no definitive mechanism for selecting between two. The decision is a judgement call. A case can be made for each and analysis have chosen to use both (thought rarely together). Perhaps the opposing results emphasize the need for caution when employing efficiencies scores for management and policies purposes and they recommend looking confirmations across viable alternatives McMillan & Chan (2004).

Mortimer (2002) makes an interesting revision of the literature comparing both methods DEA and SFA. Some interesting references of comparative analysis of both methods can be in the works of Bjurek, Hjalmarsson, Forsund (1990); Fecher, Kessler, Perelman, Pestieu (1993); Hjalmarsson, Kumbhakar, Heshmati (1996). As more recent references Wadud & White (2000); Carrigton, Coelli Groom (2002), Jaforullah & Premachandra (2003); McMillan & Chan (2004), Brazdik (2005).

In most of the mentioned works it is carried out an analysis of ranking, indicating that DMU's analyzed (universities, hospitals, insurance firms, dairy farms, firm's retailers, etc) in relation to the relative position from the group to the long of the time reaches a low consistency. Clear exceptions are for example the works of Bjurek, Hjalmarsson, Forsund (1990), Wadud & White (2000) and Jaforullah & Premachandra (2003) that report a considerable consistency in the ranking of efficiency. In this sense although this work doesn't think about as objective the analysis of the ranking of efficiency of the firms like it was already mentioned previously, the results that they were obtained they associate with the works where weak consistency exists in the comparison of the different methods.

## **2. Technical Efficiency Measures**

In the first stage of this research, we apply the nonparametric Data Envelopment Analysis (DEA) and parametric stochastic function analysis (SFA) techniques to estimate a technical efficiency.

### **2.1. Data Envelopment Analysis**

Two DEA models Charnes et al., (1978) are used in this study. A constant returns to scale (CRS) model and a variable returns to scale (VRS) model Banker et al.,(1984). The idea behind DEA is to use linear programming methods to construct a surface, or frontier around the data. Efficiency is measured relative to this frontier, where all deviations from the frontier are assumed to be inefficiency. The difference between the CRS and VRS score, as described below.

Consider  $n$  firms producing  $m$  different output using  $h$  different inputs. Thus,  $Y$  is an  $m \times n$  matrix of outputs and  $X$  is an  $h \times n$  matrix of inputs. Both matrices contain data for all  $n$  firms. The technical efficiency (TE) measure under the assumption of CRS can be formulated as follows:

$$\begin{aligned} & \text{Min}_{\Theta, \lambda} \Theta \\ \text{Subject } & -y_i + Y\lambda \geq 0, \\ & \Theta x_i - X\lambda \geq 0, \\ & X\lambda \geq 0 \\ & \Theta \in [0,1] \quad [1] \end{aligned}$$

And solved for each firm in the sample.  $\Theta_i$  is firm  $i$ 's index of technical efficiency relative to the other firms in the sample.  $Y_i$  and  $x_i$  represents the output and input of firm  $i$  respectively.  $Y\lambda$  and  $X\lambda$  are the efficient projections on the frontier. A measure of  $\Theta_i = 1$  indicates that the firm is completely technically efficient. Thus,  $1 - \Theta_i$  measures how much firm  $i$ 's inputs can be proportionally reduced without any loss in output. By adding a convexity constraint to the model above VRS is instead assumed:

$$\begin{aligned} & \text{Min}_{\Theta, \lambda} \Theta \\ \text{Subject } & -y_i + Y\lambda \geq 0, \\ & \Theta x_i - X\lambda \geq 0, \\ & N1'\lambda = 1 \\ & X\lambda \geq 0 \\ & \Theta \in [0,1] \quad [2] \end{aligned}$$

The new constraints is  $N1'\lambda = 1$  where  $N1$  is a  $n \times 1$  vector ones. This constraint makes the comparison of firms of similar size possible, by forming a convex hull of intersecting planes, so that the data is enveloped more tightly. The technical efficiency measures under VRS will always be at least as great as under the CRS-assumption.

The technical efficiency scores are computed by solving the linear program [1] and [2] for all firms in our sample and for all sample years

## 2.2. Parametric Frontier Methods

In addition to DEA, parametric techniques SFA Aigner et al., (1977) and Meusen et al.,(1977) are also used to estimate the technical efficiency in different industry with regulatory process, to test whether the manager retailer' efficiency is sensitive to the choice of the benchmarking technique. For this exercise, a translog function is used to estimate the technical efficiency of the retail industry.

The stochastic production frontier model is specified as:

$$Y_{it} = f(X_{it}; \beta) + \varepsilon_{it}$$

Where  $Y_{it}$  is the output of firm  $n$  ( $n = 1, 2, \dots, N$ ) at time  $t$  ( $t = 2, \dots, T_i$ );  $f(\cdot)$  is the production technology;  $X$  is a vector of  $n$  inputs, and  $\beta$  is the vector of unknown parameters to be estimated. The term  $\varepsilon_{it}$  is a composed error term,  $\varepsilon_{it} = v_{it} - \mu_{it}$  where  $v_{it}$  is statistical noise and  $v_{it} \sim \text{iid } N(0, \sigma_v^2)$ ,  $\mu_{it} \sim \text{iid } N(0, \sigma_u^2)$

The production technology is represented by a translog function;

$$Y_{nt} = \beta_o + \sum_{i=1}^k \beta_i x_{int} + 1/2 \sum_{i=1}^k \sum_{j=1}^k \beta_{ij} x_{int} x_{jnt} + \sum_{i=1}^k \delta_i x_{int} t + \lambda_1 t + 1/2 \lambda_1 t^2 + v_{nt} - \mu_{nt}, [3]$$

Where  $y$  and  $x$  are the natural logarithms of sales (output), purchase, employers, and fixed assets,  $t$  is the time trend. In this model technical efficiency is specified as (see Battese and Coelli (1992):

$$\mu_{nt} = \exp[-\eta(t - T_i)] \mu_i,$$

Although a translog functional form is used because it provides a second-order approximation to any arbitrary functional form. This is what is commonly termed a “flexible functional form” (Carrington et al., 2002). We used two additional methodologies Cobb-Douglas and a translog input distance function for our comparison. Following [Coelli and Perelman (1996), Coelli et al., 2003] the translog input distance function may be written as follows;

$$d_{nt} = \beta_o + \sum_{i=1}^k \alpha_i y_{int} + 1/2 \sum_{i=1}^k \sum_{j=1}^k \alpha_{ij} y_{int} y_{jnt} + \sum_{k=1}^k \alpha_k x_{int} + 1/2 \sum_{k=1}^k \sum_{j=1}^k \alpha_{kj} x_{knt} x_{jnt} + \sum_{k=1}^k \sum_{m=1}^M \gamma_{km} x_{knt} y_{mnt} + \sum_{i=1}^k \delta_i x_{int} t + \sum_{i=1}^M \phi_i y_{int} t + \lambda_1 t + 1/2 \lambda_1 t^2, [4]$$

Where  $d_{nt}$  is the log of the input distance,  $y_m$  is the log of output,  $x_k$  is the log of the  $k$ th input and Greek letters are parameters to be estimated. The input distance function must be symmetric and be homogenous of degree +1 in inputs. The restrictions required for homogeneity of degree +1 in inputs are  $\sum_k \alpha_k = 1$ ,  $\sum_{k=1}^k \alpha_{lk} = 0$  ( $k=1, 2, \dots, K$ ),  $\sum_{k=1}^k \gamma_{km} = 0$ ,  $\sum_{k=1}^k \delta_k = 0$ . Imposing the homogeneity restrictions, the efficiency score for the  $i$ th firm will be equal to  $\exp(-d_{ni})$ . We assume the distance error term has two components  $d_{it} = v_{it} - \mu_{it}$  (see Coelli and Perelman 1996).

### 3. Results

Results for all five types of efficiency are provided in table 1. In the columns 2 at 4 show up the values corresponding to the parametric models. On the down of the table 1, the statistical corresponding to the technical efficiency are shown. On the columns 2 at 4 for the parametric models, and in the columns 5 and 6 for the non parametric models BCC and CCR models.

Table 1: Estimates of models					
Variable	Dependent variable: Ln(sales) models 1 & 2; Ln(purchase) model 3				
	<i>Parametric: Stochastic frontier model</i>			<i>Non-parametric</i>	
	Cobb-Douglas (1)	Translog (2)	Dist. Func. (3)	BCC	CCR
Ln (sales)	-	-	0.719 (44.1)***	Input-oriented	Input oriented
Intercept	0.188 (13.13)***	0.069 (5.88)***	0.694 (16.2)***	Output: sales	
Ln (fixes assets)	0.030 (7.480)***	0.016 (4.80)***	0.065 (7.07)***	Input#1= fixes assets	
Ln (purchase)	0.778 (116.9)***	0.80 (119.4)***	0.060 (3.25)***	Input#2= purchase	
Ln (employees)	0.173 (24.30)***	0.166 (21.7)***	0.330 (25.0)***	Input#3= employees	
Time	0.0074 (2.19)**	0.0073 (2.6)***	0.136 (7.94)***		
Ln (fixes assets) <sup>2</sup>		0.011 (3.62)***	-0.009 (-4.2)***		
Ln (purchase) <sup>2</sup>		0.225 (26.7)***	0.060 (3.25)***		
Ln (employees) <sup>2</sup>		0.179 (16.1)***	0.356 (12.9)***		
Ln (purchase•employees)		-0.80 (-23.1)***	-0.152 (-8.3)***		
Ln (purchase•fixes assets)		-0.031 (8.54)***	0.053 (6.59)***		
Ln (employees•fixes assets)		0.012 (2.78)***	-0.066 (-6.7)***		
Ln (fixes assets)•t		0.002 (2.16)**	0.005 (2.01)***		
Ln (employees)•t		-001 (0.99)	0.035 (3.32)***		
Ln (purchase)•t		-0.006 (-3.76)***	-0.104 (-8.8)***		
time <sup>2</sup>		0.001 (0.82)	0.011 (1.93)*		
Log likelihood	874.85	1.2082.3	-269.3		
<i>Technical efficiency:</i>					
Mean	0.837	0.860	0.508	0.767	0.709
Median	0.832	0.859	0.478	0.759	0.691
Std. Dev.	0.072	0.066	0.186	0.111	0.108
Min.	0.630	0.624	0.144	0.557	0.517
Max.	0.990	0.993	0.985	1	1
Number firms per years	235	235	235	235	235

The inputs coefficients are significant at the 1% level and have the expected signs regarding economic behaviour –an increase in output is associated with an increase in the use of input. The coefficient of the time trend is positive and significant at 5% in model 1 and 1% in model 2, but the coefficients is very low, indicating a little technologic change in period.

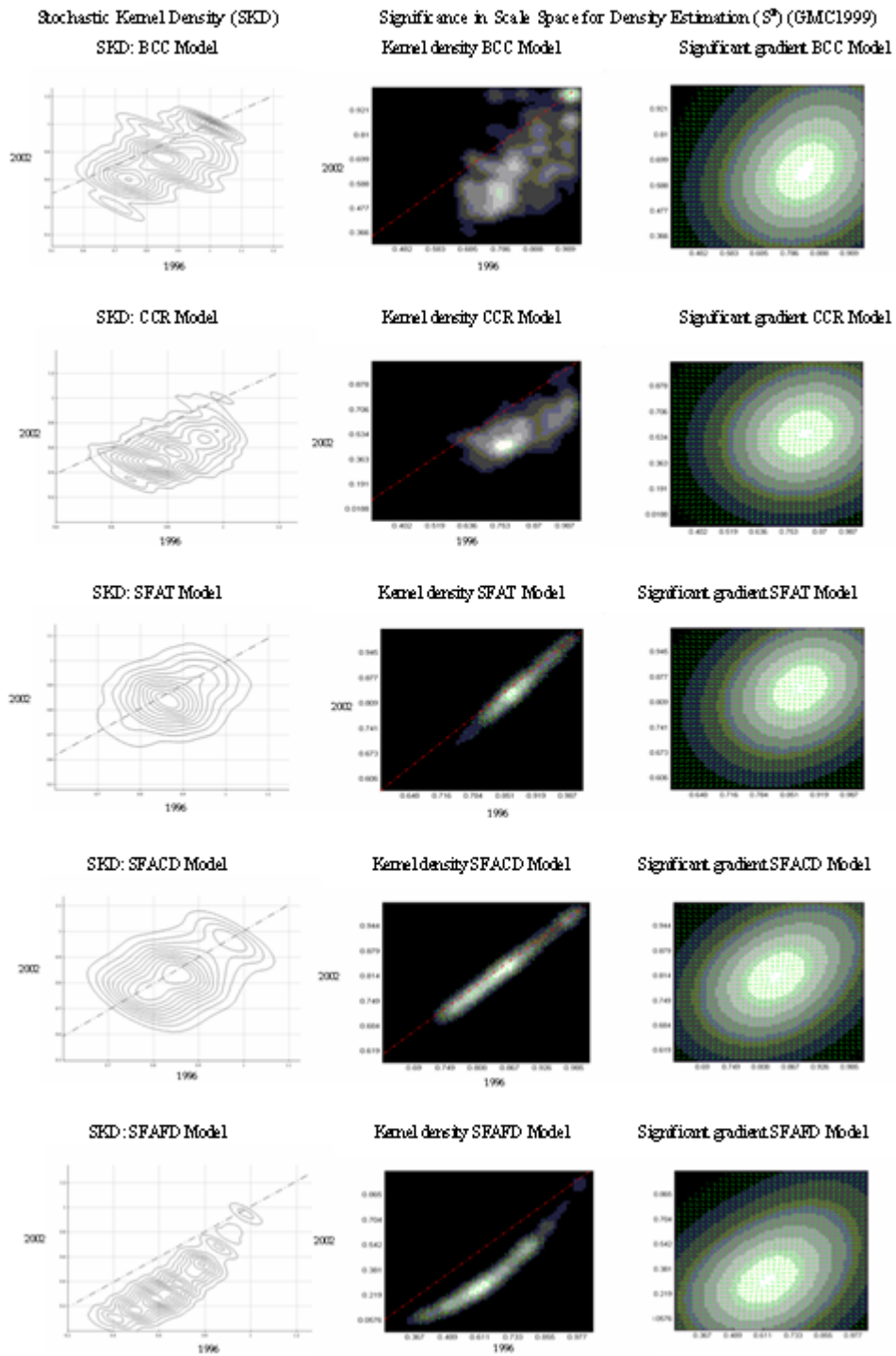
### **3.1. *Robustness check: Stochastic kernel and Sizer Tool***

In order to capture the dynamism in technical efficiency we use stochastic kernel estimations that inform about the probability of moving between any two levels in the range of values. A stochastic kernel is therefore conceptually equivalent to a transition matrix with the number of intervals tending to infinity (Quah 1997). The stochastic kernel can be approximated by estimating the density function of the distribution in a particular period  $t + k$ , conditioned on the values corresponding to a previous period  $t$ . For this we carry out a nonparametric estimation of the joint density function of the distribution at times  $t$  and  $t + k$ . An easier way of analyzing this phenomenon is shown on the left-hand side of Figure 1, which shows the contour plots, representing cuts parallel to the base of the kernel ( $X^{1996}$ - $Y^{2002}$  plane) at equidistant heights. Thus, the points are at an equal height and density.

Figure 1 report contour plots (left-hand) of the (three dimensional) stochastic kernel for our five models in relation with initial year (1996) and final (2002). Comparing the contour plots of all models there are one nuclei for high efficiency level and second nuclei in medium and low level in BCC, CCR, SFACD and SFAFD. In the model SFAT is an only nuclei in the high level of efficiency.

The negative slope diagonal in every sub-figure has a straightforward interpretation: if probably mass abandons such a diagonal, relative efficiency scores would be not the same when models efficiency are considered. In particular, if probably mass is located along the negative slope diagonal, it would indicate that level efficiency firms fall in 2002. This phenomenon takes place for all the transitions in each one of the models.

Figure 1: contour plots (left-hand) of the (three dimensional) stochastic kernel for our five models in relation with initial year (1996) and final (2002).



The firms that are in the frontier, that is to say, in the levels of high efficiency (in 1996 and 2002) they are positioned in the nuclei on the diagonal.

On the other hand Sizer is an exploratory data analysis tool, that works in conjunction with smoothing methods (see Chaudhuri and Marron, 1998). An important problem in the use of density estimation for data analysis is whether or not observed features, such as bumps are “really there”, as opposed to being artifacts of the natural sampling variability. Godtliebsen et al., (1999) propose a solution to this problem, in the challenging two dimensional case, using the graphical technique of Significance in Scale Space ( $S^3$ ). This tool is a concept from computer vision, see Linderberg (1994).

The graphics of the second column in the figure 1 show the results succeeded in using  $S^3$ . In these figures the situation is observed once concluded the process. But it is quite useful to look at the full scale space, i.e. a broad range of bandwidths. Such figures are not shown in this paper to save space. The areas change gray color to white showing the concentration of levels of efficiency. The color white is used for all region where the density is higher. In the graphics of the third column in the figure 1 the results of the maximum concentration of the efficiency appear in a more evident way.

Comparing the results of the two techniques of used evaluation Stochastic kernel and Significance in Scale Space tool to through of model different parametric and nonparametric proposed we can observe as the levels of efficiency from the initial year to the regulation process 1996 to the final year 2002 a loss of efficiency of the firms has taken place. The robustness of the results is contrasted by means of the graphics in the figure 1.

#### **4. Conclusions**

This work presents evidence of the loss of efficiency in the firms of the sector Spanish retailer experienced in the period post regulatory probably due to law of the trade retailer act of 1996. Scores are obtained from both nonparametric DEA and parametric SFA techniques.

The main contribution of this work has been the carried out evaluation of the estimates scores by means of two graphic techniques Stochastic kernel and Significance in Scale Space tool (dynamic analysis), finding an outstanding robustness in the reached results.

These results could minimize the associate risk for the management and policy when employing efficiency scores beyond the ranking analysis.



**Notes:**

The Retail Trade Act of 1996 coexists with the regional governments' exclusive responsibilities in the area. The regions define in each case what in their judgment is a large retail outlet. In 1996 the regions were authorized to award licenses essential for opening any major retail outlets, after evaluating the real needs of each locality. This does not affect the license that the local councils continue to require.

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