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Drawing and Mathematics. An Integrated Teaching

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In 2011 it was started, at the School of Architecture of Alcalá, a new course called: *Taller de Dibujo II*. Its main goal was to convey the importance of studying an architectural object from different points of view. The link would be the geometry, the coordinated subjects: design and mathematics. Teachers from both departments began an integrating task. They had two different ways of understanding teaching. It would be a subject in constant evolution. So we started an educational innovation project, which is ongoing. (UAH / EV519).

In the last ten years the importance of the parameterization has grown significantly in fields like design, engineering and architecture. Our School of architecture implemented an interdisciplinary group that was able to introduce these new skills. "The rigorous parameterization requires the assimilation of concepts much closer to mathematic geometry and software programming" (Coloma, Mesa 2012). But some experiences around the subject have put their emphasis on tools, neglecting, in our opinion, the methodological basis.

Although traditional teaching materials are not fully useful to this new subject, accumulated experiences are very valuable. Grassa-Miranda (2010) regarding the traditional teaching of geometry states that "The grammar or guiding principles of the Spanish *sistema diédrico* uses the projective schema of a model to build the student's spatial thinking, while the Anglo-Saxon direct method relies on the reconstruction of a mental image of the geometric configuration" In a similar way, we considered the importance of the object opposite to the system, or the process.

Therefore, the starting points of our methodology are the works of architecture and engineering. Objects with a complex geometry, especially those which curves and surfaces are able to be parameterized. The curve and surface become that way, protagonists of the experience. The next step is to thoroughly analyze through operations of modification and intersection.

A good analysis of a work with a complex geometry, involves the preliminary study of the project and the knowledge of the difficulties and intentions of the author. Often the most interesting geometric designs arise from the need of finding creative solutions for complex problems with the most simple and balance response, as a whole. Many of the works built by Torroja, Candela, Dieste, Maillart, Isler, Freyssinet, Frei Otto, Fisac and many others, show that the study of the object cannot be limited to the representation of form.

In this article we will show our experience and several possibilities to develop about the subject. We will describe the overall strategy and present some concrete exercises defining our scope. Finally we will propose several alternatives for further applications in future editions of the course.

Teaching of geometry. Architectural geometry.

The traditional teaching of geometry has relied on the use of abstract objects, such as points, lines, planes, boxes, cones, cylinders, etc. Maximizing the reduction of elements involved in a problem the student focuses better on general concepts. Therefore the exercises are structured so that any complex problem can be divided into several smaller difficulties, each of which are available to the student in separate procedures. This approach currently continues forming an important part of the basic teaching of geometry. Although it provides an enormous instrumental power, it develops disjointed skills and lacks a deeper insight into the architectural object as a complex geometric project. We introduce the case study as a primary means of applied knowledge.

The methodology known as problem-based learning (PBL) is suitably adjusted to the guidelines of this research. This consists on the proposition of problems as a starting point for the acquisition of new knowledge. PBL boosts self-learning, skill development, motivation, empathy and respect between individuals. It takes place in three stages: starting stage, where the prerequisites, resources and partial goals are identified, second stage, implementation, in which the partial goals lead toward the final Project, and the final stage, where conclusions are made after completing the project and the final feedback (Agudo 2010).

Taller de Dibujo II is a subject aimed to the study of geometry, at the second course of Architecture's degree. So we can count on the knowledge gained in the first year and then progress modelling complex real projects. For this we rely on computers. There is a number of software in the fields of design and mathematics that allows all kinds of modelling, but in order to use them efficiently a good knowledge of geometry is essential (Pottmann, Asperl et al. 2007).

Architecture and engineering provide us a big number of examples to study, objects of complex geometry, especially interesting are those with parameterizable curves and surfaces. About this issue we agree that "The true idea of parametric architecture is far from projects as a performance" (Viamonte, Peinado 2014) and therefore the goals of the analysis must be defined clearly, focusing on the relationship with its geometry.

Polyhedra are very interesting objects for architecture and construction, mainly because of their possibilities to be understood as a set of planes and lines. Given that simplicity, from the geometrical point of view, such objects have a lower formal complexity and therefore are less interesting than curves and surfaces, which become main characters of the experience. The great contribution of mathematical knowledge applied to this matter, is that it allows the student to determine the variables that define the forms, it is what characterizes them radically since their inception. The mathematical formulation let us identify the precise parameters of a curve or surface, and then we approach through the numerical abstraction. Mathematical geometry and programming eases rigorous approach to the study of the object (Coloma, Mesa 2012).

The next step is to thoroughly analyse through operations of modification and intersection. Once solved the basic classification of lines and surfaces, learned in previous courses of geometry, then we start to comprehend the details. The analysis descends from absolute abstraction to features that materializes the architectural or engineering object. For example a vault can be understood first as intersection between cylinders, but then we need to think about the construction process, traces, stereotomy, etc. Another example is the definition of the thicknesses that leads to the construction of a concrete shell. The model is constructed from the geometry of the object, The three dimensional model is a container of information (Garcia Reig 1999b), the levels of definition match the deepness that requires the research, from the purest abstraction to the most concrete detail.

The study of the object can not be limited to a simple reproduction of the form. Analyzing a work of complex geometry requires to study the project. In Taller de Dibujo II we start investigating about the

authors and the circumstances of the works. The collection of graphic information, both photographs and drawings, allows the students to enrich their bibliographic memory and focus their formal interests for future projects. On the other hand, the history of construction, knowledge of the people involved in the genesis and development of the project, help to understand the magnitude of the solutions (Castaño Perea, De Miguel Sánchez et al. 2014).

The most geometrically interesting designs arise from necessity. The search of solutions beyond the limits of the possible boosts the designer's capabilities and use to provide technical and artistic innovative solutions (Llorente Zurdo, De Miguel Sánchez et al. 2012). This architectural culture includes authors of great interest for their expertise on geometry and construction. Many of them are architects not sufficiently valued by the students of first courses, such as Felix Candela, Miguel Fisac and Emilio Pérez Piñero.

But the highlight part of this subject, in the Degree of Foundations, has been the recognition of the great engineers' achievements. Contributions to the field of architectural forms through the works of Eduardo Torroja, Eugène Freyssinet, Eladio Dieste, Robert Maillart, Heinz Isler, Frei Otto, etc. that act as exceptional configurations of spaces. They provide us magnificent structures that dominate gravity thanks to a strict control of their geometry and transform radically the contemporary architectural scene.

Mentoring is a fundamental support from the teacher to the student and it becomes an monitoring base for academic process . This is implemented in the initial planning of the course, based on the competencies of both, the mentor and the student. We regard mainly the skills of effective communication, conflict management, planning and teamwork (Castaño 2012).

In this article we show models developed during the past year. The order of pictures is directly related to the study of the geometric elements in mathematical classification. Thus the course began with the parametrization of plane curves then continued with curves in space and finally we take examples of surfaces, both quadrics and warped surfaces, as well as ruled surfaces and networks of curves.

Curves

The three dimensions are equally important for understanding the structures and constructions. But we can speak about linear and surface structures, thus it means that one dimension is less important than the other two, concerning their geometry. The definition of curves of the arcs is a significant source of models for the course "if the column is art, the arc is technique" (Torroja 2000). Our studies of these elements are based on the parameterization of the different curves. Anyway we do not lose sight of their constructive and structural meanings. The arc is always associated to the wall to carry loads to the foundations, besides it provides a remarkable control of the proportions. The beauty and understanding of the cycloid whose proportions are set by its definition, that refers to this mathematical concept. On the other hand, exempt arches change their shape to fit the path of the forces, so the parable is adapted to the anti-funicular curve.

The parabolic guideline in exempt arches is a simbol of modern engineering. Starting with the studies carried out by Giovanni Poleni (1683-1761) about St. Peter's dome in 1748 (Heyman 1999), through many bridges arches in the nineteenth and twentieth centuries, as well as the great Gateway arch, of Eero Saarinen in St. Louis , Missouri, completed in 1968, the power of forces has enriched the built panorama with these curves, structurally pure and attractive formally.

Fig. 1. Eugène Freyssinet, *Orly Hangars*, France

The hangars for dirigibles, made by the french engineer Eugene Freyssinet in 1923, in Orly, and destroyed in the Second World War, are a powerful example of covered space with a vault of parabolic guideline. The vault is a folded sheet of 86 meter span and barely 9 centimeters thick. The wave has a variable width from 5.4 meters at the base to 3 meters on the upper part. This impressive structure is possible thanks to its shape. It needs to neutralize its weight and resist winds, so it have to increase the inertia through the ripple of the shell. Studying this work allows us to explain the possibilities of parabolas on arches, as well as thinking about the complexity of the structural problem of a very bold solution.

Fig. 2. Norman Foster and Partners, 1999, *Dome of the Reichstag*, Berlin, Germany. Curve related to the model parameterization.

Fig. 3. Norman Foster and Partners, 1999, *Dome of the Reichstag*, Berlin, Germany.

About three dimensional curves, helix is a remarkable case. A curve associated with interesting buildings, among which highlights The Minaret of the Great Mosque of Samarra (850), the Solomon R. Guggenheim Museum in New York (1959) by Frank Lloyd Wright. And also we can study the double helix of the dome of the German parliament, by Norman Foster, completed in Berlin in 1999. This shows a transparent space, which convey a sense of dynamism. It is configured as two twisted ramps, one for going up and one for going down. The problem is that the helices are not built over a cylinder or a cone, but they are projected onto an ellipsoid, generating a space visually better controlled.

Surfaces

The ruled surfaces are very important for architecture and engineering. In addition to the cylinders, mainly linked to the intersection of their curves, we find the conoids, cylindroids, helical, etc. About them the student must understand how the guideline curves are connected, creating radically different surfaces depending on the relationship with an axis, a plan, a vertex, etc.

Fig. 4. Robert Maillart, 1939, *Cementhall Pavilion*, Zurich, Suisse. Surface developed from the model parameterization.

In 1939 Robert Maillart build a main hall pavilion at the National Exhibition in Zurich as a Portland Cement propaganda. A shell of 6 cm thick. After the fair the concrete structure was tested until failure and disappeared. In this case the shells are limited by different parabolic guidelines curves and ruled surfaces have interesting nuances depending on how they are generated.

Fig. 5. Robert Maillart, 1939, Cementhall Pavilion, Zurich

Ruled surfaces in construction provided the possibility of relying construction on straight lines. However there are many other works that use other warped surfaces that do not have these possibilities. These surfaces are understood as networks of curves in different planes. Such is the case of the Church of Atlantis, the parish of Cristo Obrero, Montevideo (1952), by Eladio Dieste. Here we can see the Vaults called Gausas. On a rectangular floor the walls and roof are ondulated. Dieste uses sinusoidal curves on the sections of the dome to shaping it as a catenary that eases the path of forces through the structural elements. Use the structural ceramics (Mas Guindal, Adell 2005), reinforcing the brickwork in two directions, taking advantage of the joints between bricks to put the rods of iron. The horizontal forces of the vaults are balanced by the steel braces inside the surface.

Fig. 6. Eladio Dieste, 1952, Iglesia de Cristo Obrero, La Atlántida, Montevideo, Uruguay. Surface developed from the model parameterization.

The geometric analysis of this work is closely related to the curves of its plan and section and the surfaces generated in the combination of both of them. Although our analysis is not structural, distribution of metal reinforcements is present in it because of the link of forms and forces through geometry.

Fig. 7. Eladio Dieste, 1952, Iglesia de la Parroquia de Cristo Obrero, La Atlántida, Montevideo, Uruguay.

One of the most complex examples that we have developed in last years is the Tribuna del Hipódromo de la Zarzuela, by Eduardo Torroja. In the section we can see a composition of different surfaces in a perfect balance. Cylindrical and toric vaults of the lower spaces are opposed to the "mainly hyperbolic" vaults of the cantilevered roof (Chías Navarro 2005). Eduardo Torroja designed this work according to the laws of statics, he modeled the structures in order to control the forces in space and always through a thorough and strict investigation of the alternatives, with a clear intellectual understanding of the problems in every meaning (García Reig 1999a).

Fig. 8. Eduardo Torroja, 1935, Tribuna del Hipódromo de la Zarzuela, Madrid. Surface developed from the model parameterization.

Torroja is an engineer and entrepreneur, son of a mathematician, represents the ideal of linking research and industry (Andrade Perdriz 1999) his work combines mathematical theory and graphical construction. Taller de Dibujo II try to learn from this mixture. There are many others structures of similar character, new opportunities to explore innovative configurations from the point of view of their geometry, proving how important it is knowledge, for projecting, building and comprehend these objects.

Fig. 9. Eduardo Torroja, 1935, Tribuna del Hipódromo de la Zarzuela, Madrid. Hyperboloid designed according to the upper face of the concrete shell.

Fig. 10. Eduardo Torroja, 1935, Tribuna del Hipódromo de la Zarzuela, Madrid.

Conclusions

This experience has reached its fifth edition. Throughout these years we have made a significant number of analysis similar to those we show in this paper, providing the students an catalogue of different cases. We have focused on operations with abstract forms and transformations. These configurations are commonly implemented in many architectural and engineering structures.

The strategy of problem-based learning has given to the students tools for geometric analysis. The managing and the level of definition of the drawings have been adapted according to the exercises. The teachers have encouraged the students to create their own geometric project, where they can schedule their learning path, with the support of the teachers according to the referred methodology.

These courses have led us to focus on making precise geometrical analysis, working with to very specific objectives. Our scope is to select the necessary geometrical information, avoiding other visual features, mainly photorealistic pictures, which according to our experience, may be a distraction for the analysis of the models.

In the medium and long term that subject is already producing interesting effects. In other areas, at the School of Architecture, the number of projects with complex geometry and parameterized elements are being multiplied.

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